

# MEASUREMENT OF THE DIAMETER OF $\alpha$ ORIONIS WITH THE INTERFEROMETER<sup>1</sup>

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## ABSTRACT

*Twenty-foot interferometer for measuring minute angles.*—Since pencils of rays at least 10 feet apart must be used to measure the diameters of even the largest stars, and because the interferometer results obtained with the 100-inch reflector were so encouraging, the construction of a 20-foot interferometer was undertaken. A very rigid beam made of structural steel was mounted on the end of the Cassegrain cage, and four 6-inch mirrors were mounted on it so as to reduce the separation of the pencils to 45 inches and enable them to be brought to accurate coincidence by the telescope. The methods of making the fine adjustments necessary are described, including the use of two thin wedges of glass to vary continuously the equivalent air-path of one pencil. Sharp fringes were obtained with this instrument in August, 1920.

*Diameter of  $\alpha$  Orionis.*—Although the interferometer was not yet provided with means for continuously altering the distance between the pencils used, some observations were made on this star, which was known to be very large. On December 13, 1920, with very good seeing, no fringes could be found when the separation of the pencils was 121 inches, although tests on other stars showed the instrument to be in perfect adjustment. This separation for minimum visibility gives the angular diameter as  $0''.047$  within 10 per cent, assuming the disk of the star uniformly luminous. Hence, taking the parallax as  $0''.018$ , the linear diameter comes out  $240 \times 10^6$  miles.

*Interferometer method of determining the distribution of luminosity on a stellar disk.*—The variation of intensity of the interference fringes with the separation of the two pencils depends not only on the angular diameter of the disk but also on the distribution of luminosity. The theory is developed for the case in which  $I = I_0 (R^2 - r^2)^n$ , and formulae are given for determining  $n$  from observations.

Table of values of  $\int_0^1 (1-x^2)^{n+\frac{1}{2}} \cos kx dx$ , for  $n$  equal to 0,  $\frac{1}{2}$ , 1, and 2, and for  $k$  up to  $600^\circ$ , is given.

It was shown in *Contributions* Nos. 184 and 185,<sup>2</sup> that the application of interference methods to astronomical measurements is not seriously affected by atmospheric disturbances, and indeed observations by these methods have proved feasible even when the seeing was very poor. The explanation of this apparent paradox lies in the fact that when the whole objective is effective, the atmospheric disturbances, being irregularly distributed over the surface, simply blur the diffraction pattern; but in the case of two isolated pencils too small to be affected by such an integrated disturbance, the resulting interference fringes, though in motion,

<sup>1</sup> *Contributions from the Mount Wilson Observatory*, No. 203.

<sup>2</sup> *Astrophysical Journal*, 51, 257, 263, 1920.

are quite distinct, unless the period of the disturbances is too rapid for the eye to follow.

When it was found that the interference fringes remain at full visibility with the slits separated by the diameter of the 100-inch mirror, it was decided to build an interferometer with movable outer mirrors (Fig. 1) in order to make tests with separations as great as 20 feet.

The interferometer beam (Plate IVa and Fig. 2) was made of structural steel, as stiff and rigid as circumstances of weight and

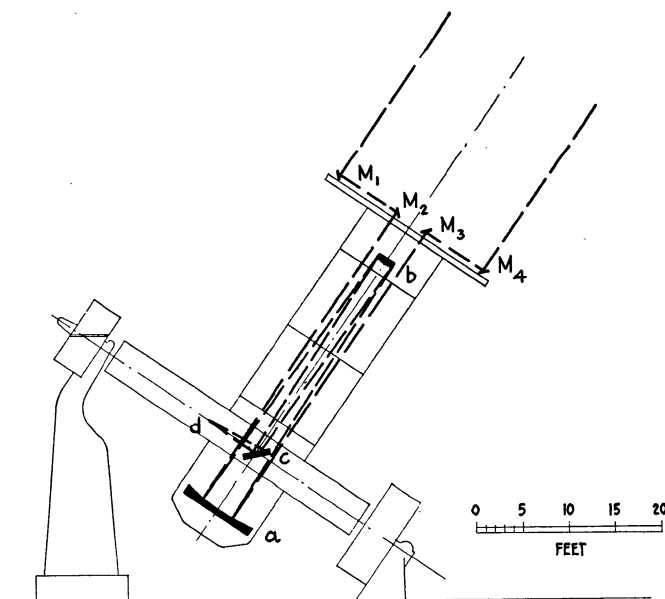


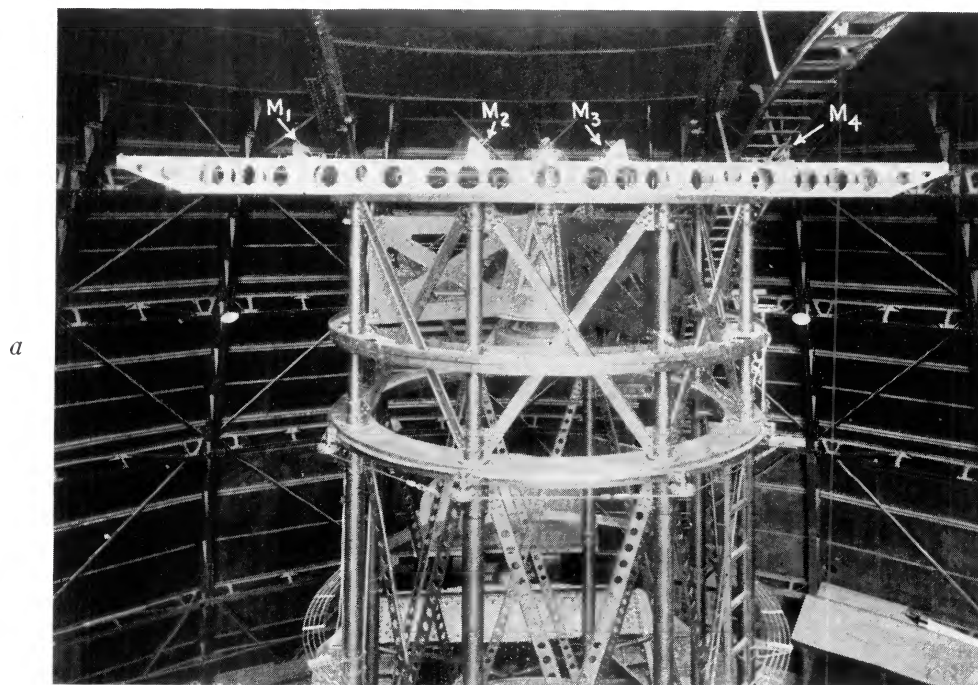
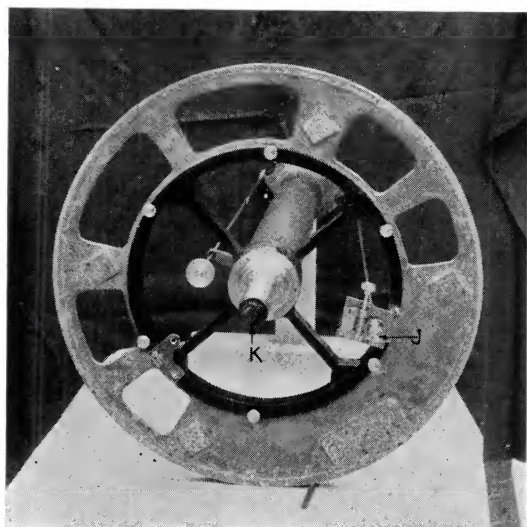
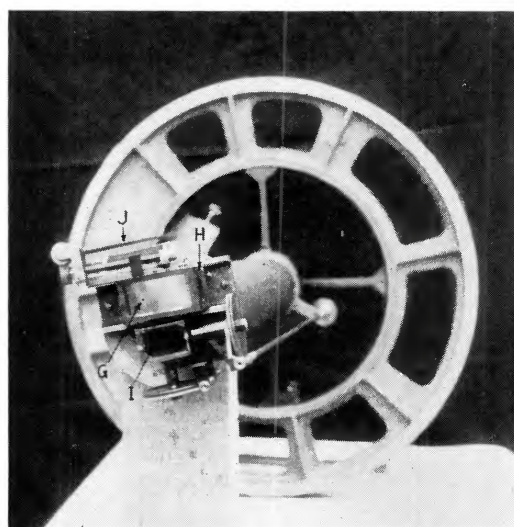
FIG. 1.—Diagram of optical path of interferometer pencils.  $M_1$ ,  $M_2$ ,  $M_3$ ,  $M_4$ , mirrors;  $a$ , 100-inch paraboloid;  $b$ , convex mirror;  $c$ , coudé flat;  $d$ , focus.

operation would permit, for flexure causes a separation of the two pencils, and any vibration as great as one-thousandth of a millimeter blurs the fringes.

The beam is constructed of two 10-inch channels with flanges turned inward, separated by pieces of 12-inch channel and covered on the bottom with  $\frac{3}{16}$ -inch (4.75 mm) steel plate ( $C$ , Fig. 2), all riveted securely together.

To reduce the weight holes were cut wherever the removal of metal would not cause a weakening of the structure. The inner edges of the top flanges were planed true to 0.001 of an inch

## PLATE IV

*a**b**c*

## INTERFEROMETER USED WITH 100-INCH HOOKER REFLECTOR

- a.* *A*, 20-foot interferometer beam on end of telescope; *M*<sub>1</sub>, *M*<sub>2</sub>, *M*<sub>3</sub>, *M*<sub>4</sub>, mirrors; *D*, Cassegrain cage  
*b, c.* Adapter at focus; *J*, rod to shift wedge; *K*, direct-vision prism; *G*, fixed wedge; *H*, movable wedge; *I*, plane-parallel compensator.

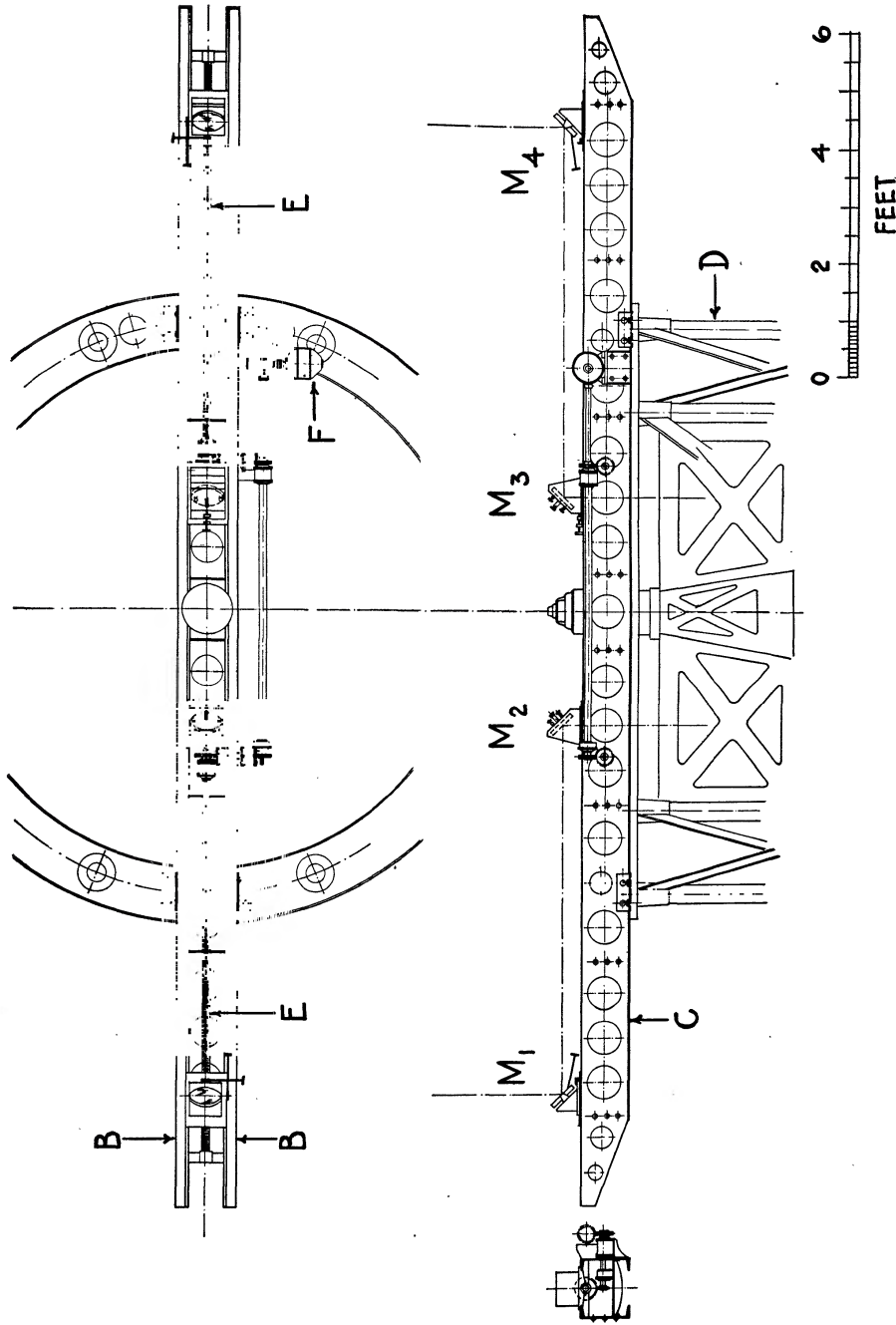


FIG. 2.—Diagram of 20-foot interferometer beam.  $M_1, M_2, M_3, M_4$ , mirrors;  $B, B$ , 10-inch channels;  $C$ , steel plate;  $E, E$ , screws to move outer mirrors;  $F$ , motor drive for screws;  $D$ , Cassegrain cage.

(0.025 mm), the beam being supported as it was to be mounted on the telescope.

Four mirrors,  $M_1$ ,  $M_2$ ,  $M_3$ ,  $M_4$  (Figs. 1 and 2 and Plate IVa), about 6 inches (152 mm) in diameter, inclined  $45^\circ$  to the base, are mounted on slides;  $M_2$  and  $M_3$  are adjusted by three screws at the back, while  $M_1$  and  $M_4$  are adjustable about two horizontal axes by means of fine screws at the ends of 9-inch (228 mm) lever arms. The mirrors  $M_2$  and  $M_3$  are permanently fixed except that  $M_3$  has a motion of several millimeters along its slide parallel to the beam.

The nearly constant separation of the mirrors  $M_2$  and  $M_3$  by about 45 inches (114.2 cm) gives the fringe pattern a fixed spacing equal to 0.02 mm, easily visible with a magnification of 1600. The mirrors  $M_1$  and  $M_4$  can be shifted along the beam; their distances from  $M_2$  and  $M_3$  must be equal, a condition satisfied as closely as possible by measurement with steel scales.

The beam is mounted on the end of the Cassegrain cage (Plate IVa and Fig. 2), which is 11 feet (3.35 m) in diameter, and all observations are made at the Cassegrain focus corresponding to an equivalent focal length of 134 feet (40.84 m). Two pencils from the star are reflected from the outer mirrors  $M_1$  and  $M_4$  to mirrors  $M_2$  and  $M_3$ , thence over the ordinary course in the telescope to the paraboloid ( $a$ , Fig. 1), the convex mirror ( $b$ , Fig. 1), the coudé flat ( $c$ , Fig. 1), and finally the focus ( $d$ , Fig. 1).

$M_2$  and  $M_3$  are adjusted during the day and  $M_1$  and  $M_4$  on a star at night; and usually after the first setting of a run the image always appears in the field of view of the telescope.

Coincidence of the two interferometer pencils at the focus is obtained by first adjusting the mirrors  $M_1$  and  $M_4$  and then tilting a plane-parallel glass plate, 15 mm thick ( $I$ , Fig. 3, and Plate IVc), in the path of one of the pencils, which also serves to compensate the path of the double wedge mentioned below.

Equality of path in the two interferometer pencils is obtained by first setting the mirrors  $M_1$  and  $M_4$  symmetrically on the beam, as nearly as possible, and then adjusting the double wedge of glass ( $G$ ,  $H$ , Fig. 3, and Plate IVc, 2 feet within the focus) in the path of one of the pencils, the relative motion of the wedges altering the path slowly and continuously.

One of the wedges ( $H$ , Fig. 3), whose angles are about  $10^\circ$ , can be moved 25 mm either side of its mean position, parallel to the inclined surfaces. One turn of the rod ( $J$ , Plate IVb) shifts this wedge 0.5 mm, thus introducing an equivalent air path of about 0.045 mm. Although fringes can be observed throughout one-third of a turn, corresponding to an air path of 0.015 mm, or about 26 light-waves, the finding of the fringes is notably facilitated by a direct-vision prism ( $K$ , Plate IVb) placed in front of the eyepiece, which permits observation of interference bands with a path-difference of several hundred waves.

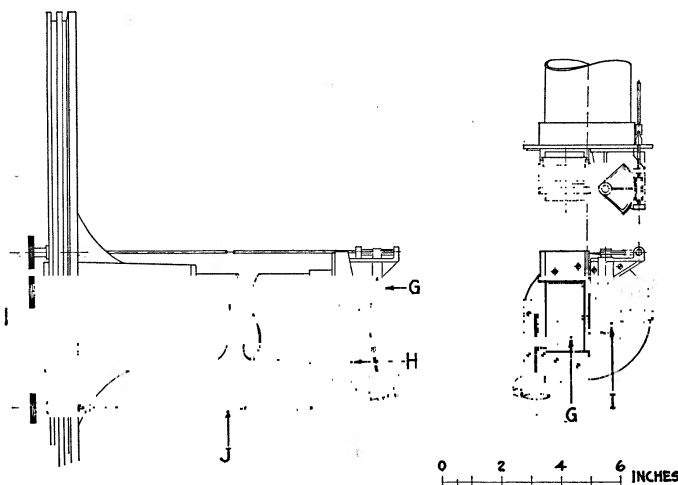


FIG. 3.—Diagram of adapter at focus.  $G$ , fixed wedge;  $H$ , movable wedge;  $I$ , plane-parallel compensator;  $J$ , rod to shift wedge.

To obtain a series of reference or “zero” fringes the end of the telescope tube is entirely covered, save for two apertures in the beam (in addition to those of mirrors  $M_2$  and  $M_3$ ), 6 inches (152 mm) in diameter. The pencils entering these apertures pass through the wedges and the compensating plate, respectively, and produce an image of the star in the field of view. When adjusted for coincidence and equality of path, these pencils interfere and produce the zero fringes which cross the reference image.

The interferometer images are next brought into the field of view of the eyepiece and made to coincide a short distance from the zero star, thus forming a second star in the field of view. Usually the adjustment of the mirrors  $M_1$  and  $M_4$  is sufficient to do this and the parallel plate compensator is used only for differential deflection

of the steel beam. As soon as the wedge is moved to equalize the path-difference in the interferometer pencils, the zero fringes disappear and count is made of the turns of the rod required to bring the interferometer fringes into view. The mirror  $M_3$  is then moved a small amount to compensate for this difference, and after several trials both sets of fringes are seen in the field of view crossing their respective images.

Differential refraction between air and glass limits the amount by which the effective thickness of the wedges may differ from that of the compensator, but interferometer fringes have been found with the wedge 20 mm from its mean position, corresponding to a 2-mm difference of air path.

Fringes were obtained with this apparatus in August, 1920, with the mirrors  $M_1$  and  $M_4$  at various distances apart, and, as there was no appreciable deterioration in their visibility at 18 feet (5.49 m) as compared with a separation of 6 feet (1.83 m), it seems reasonable to hope that this distance can be increased to 50 feet (15.24 m) or even to 100 feet (30.48 m). Work was discontinued after this series of experiments and again resumed in December, 1920.

Although provision is only just now being made for continuous alteration of the distance between the mirrors, as is essential in making accurate measurements, it was nevertheless decided to attempt a measurement of the angular diameter of  $\alpha$  Orionis, for the calculations of Eddington, Russell, and Shapley, based on estimates of apparent surface brightness, had indicated that this star would be a promising subject for investigation.

The possibility of measuring the angular diameter of a distant object by interference methods depends upon the following considerations.

Let the distribution of light in the disk be represented by

$$I = I_0(R^2 - r^2)^n,$$

$r$  being the distance from the center,  $R$  the radius of the star,  $n$  the exponent of darkening at the limb. The visibility of the interference bands is

$$V = \frac{I}{P} \sqrt{C^2 + S^2},$$

where

$$C = \int F(x) \cos kx \, dx, \quad S = \int F(x) \sin kx \, dx,$$

$$P = \int F(x) \, dx, \quad k = \frac{2\pi b}{\lambda d},$$

in which  $b$  is the distance between two pencils entering the interferometer,  $\lambda$  the mean length of the light-waves of the source, and  $d$  the distance of the source.  $F(x)dx$  is the total intensity of a strip of the source whose width is  $dx$ . For a symmetrical source such as a star disk,

$$V = \frac{C}{P}.$$

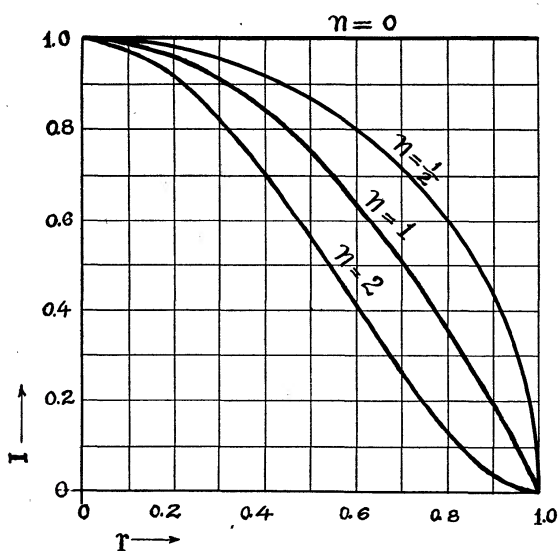


FIG. 4.—Distribution of light in various sources

If the illumination is uniform,  $F(x) = \sqrt{R^2 - x^2}$ , whence

$$V = \frac{\int_0^R \sqrt{R^2 - x^2} \cos kx \, dx}{\int_0^R \sqrt{R^2 - x^2} \, dx},$$

tables for which have been computed by Airy.

If the illumination can be represented as a function of the distance from the center,  $I = (R^2 - r^2)^n$ , then



$$F(x) = \int_0^{\sqrt{R^2-x^2}} (R^2-x^2-y^2)^n dy$$

which, expanded in series and substituted in  $V$ , gives

$$V = \frac{\int_0^R (R^2-x^2)^{\frac{2n+1}{2}} \cos kx dx}{\int_0^R (R^2-x^2)^{\frac{2n+1}{2}} dx}$$

This integral has been computed under the direction of Professor Moulton, the results being summarized in the accompanying

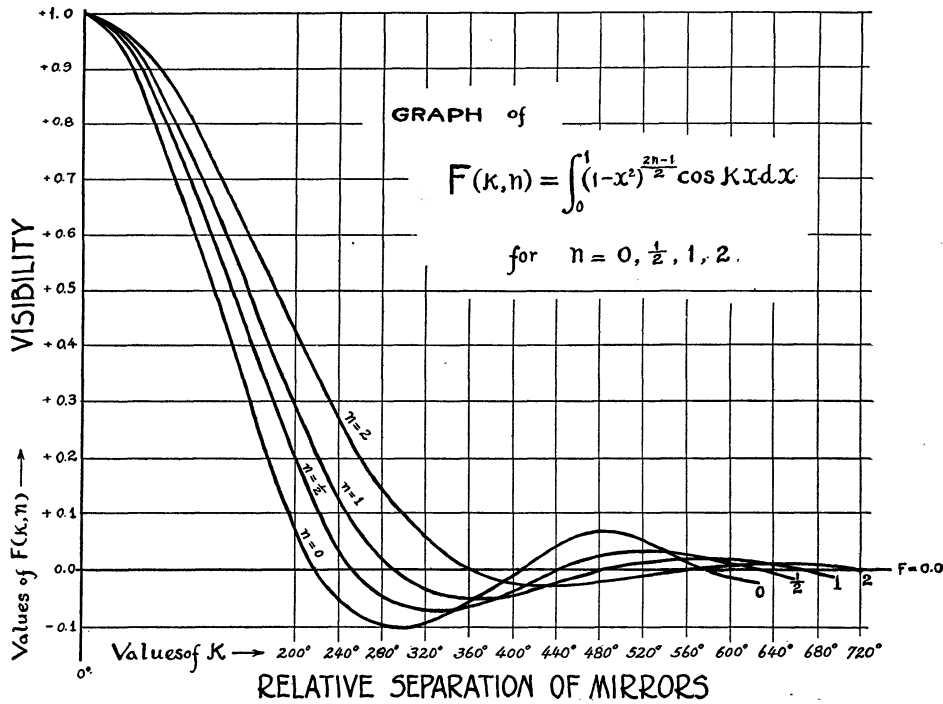


FIG. 5.—Visibility-curves for various sources

table; and in Figure 4 is shown the distribution of light in the source for the values  $n=0, \frac{1}{2}, 1$ , and  $2$ , and in Figure 5 the resulting visibility-curves.

It will be noted that for  $n=0$  the first disappearance of the fringes occurs at  $k=220^\circ$ , while for  $n=0.5$ , the value is  $k=257^\circ$ . For this value of  $n$ , the corresponding value of the diameter of Betelgeuse would be about 17 per cent greater than given below. (For the sun, the value of  $n$  is not far from 0.4.)

## VALUES OF

$$F(k, n) = \int_0^1 \frac{x^{2n+1}}{(1-x^2)^2} \cos kx dx$$

$n=0$		$n=0.5$		$n=1$		$n=2$	
$k$	$F(k,n)$	$k$	$F(k,n)$	$k$	$F(k,n)$	$k$	$F(k,n)$
0°	+0.785	0°	+0.785	0°	+0.785	0°	+0.785
100	+0.507	30	+0.765	40	+0.746	40	+0.761
130	+0.378	60	+0.702	80	+0.663	80	+0.694
160	+0.243	90	+0.607	120	+0.536	120	+0.590
200	+0.065	120	+0.490	160	+0.383	160	+0.468
230	-0.024	150	+0.363	200	+0.237	200	+0.342
240	-0.050	180	+0.238	240	+0.112	240	+0.221
280	-0.100	210	+0.127	280	+0.024	280	+0.123
320	-0.095	240	+0.038	320	-0.029	320	+0.054
360	-0.053	257°27'	0.000	360	-0.045	360	+0.003
400	-0.007	270	-0.024	400	-0.039	400	-0.019
440	+0.036	300	-0.057	440	-0.020	440	-0.024
520	+0.042	330	-0.068	480	-0.001	480	-0.018
600	-0.011	360	-0.059	520	+0.012	520	-0.006
.....	.....	390	-0.040	600	+0.015	560	0.000
.....	.....	420	-0.016	640	+0.005	600	+0.005
.....	.....	450	+0.005	680	-0.004	680	+0.005
.....	.....	480	+0.019	.....	.....	720	+0.002
.....	.....	510	+0.028	.....	.....	.....	.....
.....	.....	540	+0.026	.....	.....	.....	.....
.....	.....	570	+0.019	.....	.....	.....	.....
.....	.....	600	+0.009	.....	.....	.....	.....
.....	.....	630	-0.002	.....	.....	.....	.....

While it may be too much to expect that we shall be able to deduce the actual distribution of the light in the source from observations of the visibility-curve itself, it may be worth while to point out that this is theoretically possible; and perhaps the case will not be entirely hopeless when the requisite skill in making the observations has been obtained by practice.

If  $b_1$  and  $b_2$  are the distances at which the fringes vanish for the first and second times, then the following formula will give a fair approximation to the value of  $n$  in the light-curve:

$$n = -1 + 75 \left( \frac{b_1}{b_2} - \frac{1}{2} \right)^2.$$

Or again, if  $V_m$  is the visibility at the first negative maximum (that at  $b=0$  being unity),

$$n = 0.22 \left( \frac{1}{V_m} - 7.8 \right)^{0.7}.$$

It will be seen from this discussion that the measurement sought is that value of the separation of the outer mirrors  $M_1$  and  $M_4$  for which the fringes vanish. This corresponds to the point where the graphs (Fig. 5) cross the horizontal axis, that is, for the value  $F(k,n)=0$ . A preliminary investigation by Merrill, with the apparatus used by Anderson<sup>1</sup> in the measurement of Capella, revealed in fact a definite decrease in visibility of the fringes of  $\alpha$  Orionis for the maximum separation of the slits (100 inches). This was true, moreover, for all position angles, which indicated that the star is not a binary and that the decrease in visibility is to be attributed to a measurable diameter.

On December 13, 1920, after preliminary settings on  $\beta$  Persei with the mirrors separated 81 inches (229 cm) and on  $\beta$  Persei and  $\gamma$  Orionis with a separation of 121 inches, thus insuring that the instrument was in perfect adjustment, it was turned on  $\alpha$  Orionis and fringes across the interferometer image were sought for some time, but could not be found. The seeing was very good, and the zero fringes could be picked up at will.

When next turned on  $\alpha$  Canis Minoris the fringes stood out on both images with practically no adjustment of the compensating wedge, which furnishes a check on the disappearance of the fringes for  $\alpha$  Orionis.

It is clear from these observations that the disappearance of the fringes in the case of  $\alpha$  Orionis cannot have been due to any disturbance of the mirrors caused by changes in the position of the telescope, for changes such as those here involved require only a few turns of the rod controlling the compensating wedge to counteract differences in atmospheric path and flexure and bring the fringes again into view. There is therefore no chance that they were simply overlooked in the case of  $\alpha$  Orionis.

To observe the recurrence of the fringes near the second maximum for  $\alpha$  Orionis, the mirrors were next separated a distance of 13 feet and the telescope turned on  $\alpha$  Canis Minoris, fringes appearing across both star images as soon as the component pencils were brought into coincidence. On December 14 and the nights following, the seeing was very poor, the visibility of standard check stars

<sup>1</sup> *Mt. Wilson Contr.*, No. 185; *Astrophysical Journal*, 51, 263, 1920.

being decidedly less than on the night of December 13; consequently no attempt was made to observe the second maximum and minimum of  $\alpha$  Orionis, but attention was turned to other stars, and settings were made on  $\alpha$  Ceti,  $\alpha$  Tauri, and  $\beta$  Geminorum. The zero fringes were seen in every case, although with slightly decreased visibility; but it was only at rare intervals of better seeing that the interferometer fringes could be seen at all. From this fact it is presumed that there was an actual decrease of visibility for these stars and that some estimate of their diameters can be made with the 20-foot beam. Observations on these and other objects of presumably large diameter are being continued.

Assuming that the effective wave-length for  $\alpha$  Orionis is  $\lambda$  5750, its angular diameter from the formula  $\alpha = 1.22 \lambda/b$  proves to be  $0''.047$ ; and with a parallax<sup>†</sup> of  $0''.018$  its linear diameter turns out to be  $240 \times 10^6$  miles, or slightly less than that of the orbit of Mars. This value corresponds to a uniformly illuminated disk, while for one darkened at the limb, this result, as mentioned above, would be increased by about 17 per cent. The uncertainty of the measurement of the angular diameter is about 10 per cent.

Cordial acknowledgment is tendered to Director George E. Hale for placing the resources of the Observatory at our disposal and for his enthusiastic co-operation in furthering the investigation.

Mr. J. A. Anderson was present on several occasions and we wish particularly to acknowledge his valuable assistance in checking the measures on December 13.

MOUNT WILSON OBSERVATORY  
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<sup>†</sup> The weighted mean of Adams' spectroscopic parallax,  $0''.012$ , and the trigonometric values of Elkin,  $0''.030$ , and of Schlesinger,  $0''.016$ .