VANDE SPIEGHELING DER SINGCONST

ON THE THEORY OF THE ART OF SINGING

EDITED BY

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INTRODUCTION
SIMON STEVIN'S VIEWS ON MUSIC

Simon Stevin for some time seems to have contemplated writing a treatise on music. If ever he accomplished this design, the work must obviously have been lost. Only some fragments were discovered in 1884 by D. Bierens de Haan in a collection of miscellaneous manuscripts, which belonged to Constantijn Huygens (1596-1687), the well-known secretary to the Princes of Orange, who, at the same time, was a gifted poet and musician. (*Cf.* this edition, Vol. I, p. 33, work XV). This collection, now in the possession of the Koninklijke Nederlandse Akademie van Wetenschappen at Amsterdam, is preserved at the Koninklijke Bibliotheek at The Hague, The Netherlands.

Stevin often divided his books into a main part, containing the established doctrine, and an appendix dealing with controversial matters, in order "not to obscure the instruction by dispute", as he says. Accordingly we find two main parts, or sketches thereof, and two appendices. Neither of these treatises is complete, and in the plan they show an appreciable difference in the stress laid on certain points. One draft is in Stevin's own handwriting. It dwells rather more upon discussions than the other. This part will be reproduced hereafter. The other draft, which has been copied as if in preparation for print, indulges rather more in elementary definitions. It shows some gaps, presumably to be filled up later.
Stevin used to open his books with a summary. In one of the drafts the main part and the appendix contain pages bearing the title *cort begrijp* (*i.e.* summary), but one of these pages is blank, and the other contains a dedication to the "singing masters" of his time and the statement that he will give his critical remarks in an appendix.
Nowhere does Stevin use the word *music*. He always writes *singing*. Composers are called makers of singing. The stave is called singing ladder. Perhaps the word *singconst*, the art of singing, was the best translation into Dutch he could think of for *musica*. As is well known, Stevin was extremely keen in inventing and propagating vernacular translations of Latin words (see Vol. I, p. 6 and p. 58). It is to the semantic power of the Dutch language in making a word express its meaning properly by means of its components, and to the lack of this power of the Greek language, that he ascribes the fact that the clever Greeks failed to find the correct solution of how properly to divide the string to suit the true musical scale, whereas he himself was able to offer this solution.

For his reflections and conclusions Stevin based himself on "natural singing" (*natuerlicke sanck*), taking for granted that natural singing is an empirical fact liable to be observed with an amount of reasonable exactitude sufficient for all kinds of practical purposes. That, of course, is not rigid mathematical precision. The scale of natural singing shows five major steps and two minor steps. Stevin maintains that all major steps must be equal. So are the minor steps, each being one half of a major step. Thus, the sum of all steps in a "round" (*ommeganck;* we call it octave) amounts to six major steps. The major steps are whole tones, the minor steps are semitones.

With this statement Stevin took sides in the controversy on the problem of how to place frets on the fingerboard of a stringed instrument in order to ensure the correct intervals. The problem arises from the recognition that two fundamental intervals are equally important, but at the same time clash owing to their mutual incommensurability. These concordant intervals are commonly called the perfect fifth and the perfect major third. Though a certain amount of musical training will be helpful in understanding the nature of the difficulty, the general reader might grasp the point at issue as follows, if he does not mind being confronted with figures.

It is quite natural to divide a string of a lute, between neck and bridge, into two equal parts, and to listen to the sounds produced. Again, it is natural to divide the halves into two. It is also natural to divide the string into three equal parts, and again each third part into two, and into three. Thus, taking the whole length to have 144 parts, we get a division at the numbers

144	128	120	112	108	96	80	72	64	48	36	32	24	16

For shorter parts of the string we get higher notes.

We place frets on the fingerboard according to this division. By pressing down the string on these frets, we can easily produce the notes required.

We may use the numbers given to designate the notes thus produced. Actually, however, people have agreed to call them by letters, *e.g.* the following

A	*B*	*c*	.	*d*	*e*	*g*	*a*	*b*	*e'*	*a'*	*b'*	*e''*	*b''*

The note 112 was not included in the ancient lettered system. For our purpose we may ignore it.

The most important relation is the interval between the notes corresponding to a certain length of string and its half. Such notes are designated by the same letters, as *A* and *a* (144 : 72), or *a* and *a'* (72 : 36); as *B* and *b* (128 : 64), or *b* and *b'* (64 : 32) and *b'* and *b''* (32 : 16); again *e* and *e'* (96 : 48), or *e'* and *e''* (48 : 24). Their relation, their interval (2 : 1), is called an octave.

Next comes the so-called interval of the perfect fifth, 3 : 2, as between *A* and *e*, *e* and *b*, *c* and *g*, *d* and *a*, *a* and *e'*. Then follows the interval of the perfect fourth, 4 : 3, as between *A* and *d*, *d* and *g*, *B* and *e*, *e* and *a*, *b* and *e'*, *e'* and *a'*, *b'* and *e''*. There are the intervals of the major third, 5 : 4, as between *c* and *e*, *g* and *b*.

For the technique of playing it is desirable that the fingers should not have to reach out far from the neck to the smaller numbers. For that reason a second string will be provided, with the same length as the first, a lighter one which at full length produces the note 108, which we called *d*. The second string at full length producing the note 108, the frets will produce notes corresponding to numbers proportionally reduced in the ratio $^3/_4$. Here they are.

108	96	90	81	72	60	54	48	36	27	24	18	12

The names by letters will be

d	*e*	*f*	*g*	*a*	*c'*	*d'*	*e'*	*a'*	*d''*	*e''*	*a''*	*e'''*

There is a gain. A new note represented by a new letter, *f*. But there is a clash also. The fret number 108, producing by the first string a note *d*, is now producing a note $g = 81$, which cannot be the same note as $g = 80$ played on the first string.

Using a third string, which at full length produces the note 81 = *g*, again there appears a new note, but we find more clashes, as is seen from the table below of the notes given by numbers and by letters.

Table

144	128	120	108	96	80	72	64	48	36	32	24	16
108	96	90	81	72	60	54	48	36	27	24	18	12
81	72	$67^1/_2$	$60^3/_4$	54	45	$40^1/_2$	36	27	$20^1/_4$	18	$13^1/_2$	9
A	*B*	*c*	*d*	*e*	*g*	*a*	*b*	*e'*	*d'*	*b'*	*e''*	*b''*
d	*e*	*f*	*g*	*a*	*c'*	*d'*	*e'*	*a'*	*d''*	*e''*	*a''*	*e'''*
g	*a*	*b-flat*	*c'*	*d'*	*f'*	*g'*	*a'*	*d''*	*g''*	*a''*	*d'''*	*a'''*

There is a note *c'* = 60 $^3/_4$ clashing with *c'* = 60 and *c* = 120 on the second and first strings. There are notes *g'* = 40 $^1/_2$ and *g''* = 20 $^1/_4$ on the third string clashing with *g* = 80 on the first string.
These clashes constitute a very serious difficulty in playing on a lute tuned in this way. The error 81 : 80 is called a "comma".

J. Murray Barbour, in his book on *Tuning and Temperament* (Michigan State College Press, 1953), presents a historical survey of the attempts to find a satisfactory solution for the problem how to improve the placing of the frets. His book contains an ancient explanatory picture of a lute, by Gassani, indicating the places of the frets.
It shows a division equivalent to the division given above, making in numbers
72 64 60 54 48 40 36
Gassani adds some more, filling up gaps
72 68 64 60 57 54 51 48 45 40 36
We can fill up the whole tones 45 : 40 : 36 in this way:
45 42 40 38 36
We now see three series of semitones
72 : 68 : 64 : 60 = 18 : 17 : 16 : 15
60 : 57 : 54 : 51 : 48 : 45 : 42 = 20 : 19 : 18 : 17 : 16 : 15 : 14
42 : 40 : 38 : 36 (: 34 : 32 : 30) = 21 : 20 : 19 : 18 (: 17 : 16 : 15)
I continued the last series beyond 36 as a repetition of the first, lower series.
There is a continuous range of semitones with the values 14 : 15 : 16 : 17 : 18 : 19 : 20 : 21, from major semitones 14 : 15 to minor semitones 20 : 21.
Vincenzo Galilei (*Dialogo della musica antica e moderna*, Firenze, 1581) disclaimed such a variety of semitones. Before Stevin, he wanted them all to be equal, and he chose the value 17 : 18, which happens to be midway. For the fifth, seven such semitones added would make $18^7 : 17^7$, about 6.12 : 4.10 (canceling 10^8), a rather good approximation to 6.15 : 4.10 = 3 : 2. The defect is 3 in 615, or 1 in 205.
For the major third, four such semitones would make $18^4 : 17^4$ = about 10.50 : 8.35 (canceling 10^4). This is a poorer approximation to the accepted value 5 : 4 = 10.50 : 8.40. The excess is about 5 in 840, or 1 in 168.
Now the octave, when taken to consist of twelve such semitones, turns out to be $18^{12} : 17^{12}$, approximately 11.57 : 5.83 instead of 11.66 : 5.83 = 2 : 1. The defect is 9 in nearly 1170, all but 1 in 130. It is three fifths (and in the opposite sense) of the comma excess of 1 in 80 in the values 81 and 80, 60¾ and 60, noted earlier.

The equal temperament by semitones 17 : 18 distributes the error of the comma over the octave ($-^3/_5$ c), the fifth ($-^2/_5$ c) and the major third ($+^1/_2$ c).
Vincenzo Galilei's rule seems to have been commonly accepted at the end of the sixteenth century, as it is to this day, but only for the lute, the viol, and similar instruments.

For organs and for harpsichords no attempt was made to divide the octave into twelve equal steps. Organists tried to have pure octaves and perfect major thirds by a slight adjustment of the fifth. They corrected the sequence of four fifths

$$486 : 324 : 216 : 144 : 96$$

so as to have perfect consonance between 480 and 96, because

$$480 : 96 = 5 : 1.$$

The comma excess 486 : 480 = 81 : 80 is distributed over the four steps, each step losing one fourth of a comma, *i.e.* 1 in 320, as follows (approximately):

$$480 : 321 : 214^2/_3 : 143^5/_9 : 96.$$

In order to have perfect major thirds (as between $480 = 5 \times 96$ and $384 = 4 \times 96$), a small infringement is thereby made of the perfect fifths.
Barbour (*l.c.*, p. 26) gives the credit for the first description of this method of tuning to Pietro Aron (*Toscanello in musica,* Venice, 1523). It is the mean-tone temperament, strongly advocated by Gioseffo Zarlino (1517-1590; Stevin occasionally calls him *Meester Sarlijn*) and by Francesco Salinas (1577), two great early legislators of music. It has been in use for three centuries.

Stevin boldly did away with all these subtleties. In his view, all semitones had to be equal. In this he agreed with Vincenzo Galilei, that dissenting pupil of Zarlino.
He rejects any relationship between concordant intervals and ratios of integer numbers. For him the numbers resulting from the division of the ratio 2 : 1 into twelve equal ratios, twelve times the twelfth root of 2, are the *true* numbers. Barbour's remark is very appropriate (*l.o.*, p. 7): "In his days only a mathematician (and perhaps only a mathematician not fully cognizant of contemporary musical practice) could have made such a statement." Barbour adds: "It is refreshingly modern, agreeing completely with the views of advanced theorists and composers of our day." [1]) It is Stevin's outstanding achievement that he produced the exact proportional numbers, between 10 000 and 5 000, in four figures, representing the steps of twelve semitones in the octave leading from 1 to $^1/_2$. He was able to do so, referring to his French work on arithmetic (1585, this ed. Vol. II. B) where he had shown that the requirement of twelve equal ratios leading from 2 to 1 involves the twelfth root of 2. By combining the operations of computing two square roots and subsequently a cube root, he finds for the twelfth root of 2 the ratio 10 000 : 9 438 = 1.0595 : 1. The more exact figure is 1.059463 . . . . .
Stevin never mentions the approximate value $^{18}/_{17}$, familiar to makers of lutes, who used it in fixing frets on the fingerboards.
He had no bump for the plain simplicity of small integer numbers. In his treatise on arithmetic (Work V) he had explained that there are "no absurd, irrational,

[1]) The present editor believes that Stevin's duodecimal division of the octave is now going to be superseded by the division into 31 steps, advocated by Nicola Vicentino (1588) and Christiaan Huygens (1691).

irregular, inexplicable, or surd numbers" (see this edition, Vol. II B, p. 532, also Vol. I, p. 23).

For him a number like $2^{7/12}$ is as good as any other, say $^3/_2$. If anybody should doubt that the sweet consonance of the fifth could be compatible with so complicated a number, then, says Stevin, rather haughtily and aggressively, he is not going to take pains to correct the inexplicable irrationality and absurdity of such a misapprehension. He repudiates the Pythagorean values for the intervals ($^3/_2$ for the fifth, $^9/_8$ for the second, $^{81}/_{64}$ for the major third, $^4/_3$ for the fourth) on the ground that they lead up to the ratio $^{256}/_{243}$ for a semitone (the minor limma). This, when subtracted from a whole tone ($^9/_8$), leaves another semitone with a ratio very close to $^{256}/_{240}$. Stevin remarks that this major semitone is all but a quarter larger than the previous minor semitone (the differences of 243 and 240 from 256 being 13 and 16 respectively). All semitones having to be equal, the initial assumption of $^3/_2$ for the ratio of the fifth must be wrong. For Stevin the equality of the twelve semitones follows from the fact that in tuning a harpsichord one obtains a closed cycle of fifths and fourths. Strictly speaking, the excess of twelve fifths over seven octaves should be 1 part in 73 (comma of Pythagoras). Stevin, however, ascribes any small deviations from the perfect cycle to unavoidable experimental errors.

Joseph Needham, in Vol. 4, Part 1, of his *Science and Civilisation in China*, refers to the duodecimal equal temperament as "the princely gift of Chu Tsai-Yü". He points out that at the end of the 16th century there was a great flow of Chinese information into Europe. He urges the probability of some idea of Chu Tsai-Yü's solution having floated towards Stevin's mind. Stevin himself refers to Prop. 45 in his book on arithmetic as the source of his method for finding the 12 equal semitones, ascribing his success to the wonderful semantic power of his Dutch language. He could not have said so, if he had to admit that a Chinese had been able to find the formula without Dutch words. The book of Chu Tsai-Yü quoted by Needham is dated 1584. Stevin's book on arithmetic appeared in 1585. We can agree with Needham saying "the name of the inventor is of less importance than the fact of invention." As far as we know Stevin, we can apply to him the very same words of praise which Needham gives to Chu Tsai-Yü: "Stevin himself would certainly have been the first to give another investigator his due, and the last to quarrel over claims of precedence".

There is the ancient problem, come down from the Greeks, as to whatsoever sounds may have to do with numbers. In Stevin's time people had no clear consciousness of the frequency of vibrations. He speaks of "coarseness" or "fineness" determining pitch, and postulates a proportionality of this coarseness to the length of the sound-producing part of a string. By way of example, he refers to the half, to the quarter, and to the eighth part of a string only. He does not mention other aliquot parts, or $^2/_3$, or $^3/_4$ of a string as examples. In this he shows a bias against integer numbers. Two is the only integer admitted by him in music. One would not have expected such a bias in a mind which knew quite well that the regular solids exhibit only selected integers in the number of their faces, edges, and angular points. Perhaps he would have admitted that consonant intervals, and their beauty, primarily have to do with integer numbers if he could have seen Lissajous' delightful figures of interfering oscillations. He never mentions the phenomenon of beats, so essential for tuning perfect concords. Stevin never verified whether on a harpsichord tuned with a closed cycle of fifths and fourths the thirds and sixths would turn out to be concords. They certainly would not! Nevertheless he takes the consonance of these intervals for granted as an empirical fact. He decides rather by definition which intervals are good and which are bad.

As a practical rule, the "singing masters" condemned the interval of the fourth in polyphonic singing. This interdiction is not recognized by Stevin. He argues that very often, when one hears two instruments, *a* and *b*, playing in unison, it is very difficult to know whether they are playing at the same pitch or one octave apart. If a third instrument, *c*, plays in consonance with both, then of course it is in consonance with each of them. In case the concord seems to be that of a fifth, it is difficult for the ear to decide whether *c* makes a fifth with both *a* and *b*, or with one of them only, making a fourth with the other. But, this being so, the fourth must be a good concord too.

Stevin refuses to recognize a difference in singing with a flat on the stave or without (*mollaris* and *duralis*). He says that by transposition every tune can be written on the stave without a flat. In this he is right. Of course this has nothing to do with the difference in mode, with minor and major scales. There is no chapter on this subject of modes, but we have collected some scattered data.

Sometimes, in the scale the note *si* is flattened by a semitone to *sa*. Stevin seems to have seen a reason for giving *sa* a place on the stave without the sign for a flat. It is curious to see that in certain diagrams he assigns the vocables

ut re mi fa sol la sa ut

to the letters

g a b c d e f g

If he had assimilated *ut* to *c*, as we do, of course *sa* would have meant *b*-flat, and *si* would have to be *b*-natural (the Germans would say *b* and *h*, respectively). In one place Stevin promises to return to this question of *sa* and *si*, but no chapter on this question is included. In the manuscript there is no consistent notation of *sa* and *si* on the stave.

We do not know whether Stevin ever considered his work to have been brought to a satisfactory conclusion, and whether he intended to publish it. It might well be that discussions with musicians made him change his mind in some respect. Among the manuscripts of Constantijn Huygens mentioned above, published as an appendix to Stevin's *Singconst* (listed as Work XV in Vol. I of this edition, p. 33), there is a letter to Stevin from Abraham Verheyen, organist at Nijmegen (Gelderland), who urges that experiment, in tuning a harpsichord, shows that the three major thirds, *i.e.* six whole tones, do *not* make an octave. He explains to Stevin the merits of the current mean-tone temperament, and how to compute the ratios involved. Verheyen also produces an example of a song in two parts, clearly showing the difference of major and minor semitones. We know that Isaac Beekman (1588-1637, *Journal*, ed. C. De Waard, The Hague, 1942, Vol. 4, p. 157) at first very much admired Stevin's proportional division of the octave. Later he rejected it.

Maybe the criticisms of very able friends shook Stevin's sturdy conviction a little, so that he abandoned the idea of making a full size treatise based on his mathematical axiom.

VANDE SPIEGHELING DER SINGCONST

BEPALINGHEN

1e *Bepaling*

Trap is de naeste vervolghende climming diemen inde nateurlicke sanck rijst wiens minste stemming cleentrap geheeten wort de meeste groote trap.

2e *Bepaling*

Nateurlicke sanck is, die deur oirdentlicke climming aldus gheschiet: Twee groote trappen een cleene drie groote een cleene twee groote een cleene drie groote een cleene. Ende soo overhandt oirdentlick voort.

Verclaring

Anghesien dat de leeck luijden sonder kennis vant onderscheyt tusschen halve ende heele trappen door nateurlicke gheneghenheijt sulcken voortganck ghebruyken soo wortse met goede reden nateurlicke sanck ghenoemt want 2 of 3 halve trappen ofte 4 of 5 heele trappen vervolghens achter malcander te singhen en is niet alleen moeyelick om doen maer oock int anhooren onbehaghelick ende als onnateurlick.

3e *Bepaling*

Dese seven trappen na de nateurlick gesanck oirdentlick climmende maken des ghesanckx een ommeganck.

Verclaring

Wanneer men boven een ghestelde toon seven trappen rijst met oirdentlick climming soo heeft het laetste gheluijt sulcken ghelyckheijt mettet eerste dattet schijnt al ofmen een ommeganck ghedaen hadde ende wederom quaem daermen begosten: Inder voughen datmen sulcx van weghen die ghelyckheyt ommeganck heet: Welck ghenouchsaem toegaet als inde Sterreconst met de slangkeeren, die de Maen duer haer daghelicx roersel daghelicx beschryft welcke eyghentlick gheen evewijdighe ronden sijnde nochtans om de ghelijckheyts wille alsoo ghenoemt worden.

4e *Bepaling*

Die seven trappen worden elck aldus ghenoemt, *ut, re, mi, fa, sol, la, si*, wiens trappen van *mi* tot *fa* ende van *si* tot *ut* cleen sijn dander al groot.

5e *Bepaling*

Twee gheluyden even hooch sijnde hun verlycking wert selftoon ghenoemt. Maer een cleen trap verschillende half toon. Een groote trap verschillende toon: Een groote met een cleene verschillende anderhalftoon: Twee groote verschillende tweetoon. Ende soo oirdentlick voort.

ON THE THEORY OF THE ART OF SINGING[1])

DEFINITIONS

1st *Definition*

Step is the next subsequent ascent which one rises in natural singing, of which the smaller variety is called minor step, the larger, major step.

2nd *Definition* [2])

Natural singing is that which by an orderly ascent takes place as follows: two major steps, one minor, three major steps, one minor, two major steps, one minor, three major steps, one minor, and so on gradually, in orderly sequence.

Explanation

Since lay people, not knowing the difference between half steps and whole steps, use this progression by natural inclination, it is with good reason called natural singing, because singing two or three half steps, or four or five whole steps in succession not only is difficult to accomplish, but also is unpleasant to hear as well as unnatural.

3rd *Definition*

These seven steps, ascending in an orderly way according to natural singing, make one round of singing.

Explanation

When one rises seven steps above a given note in an orderly ascent, the last sound is so similar to the first that it seems as if one had made a round and arrived again at where one began, so that, because of that similarity, this is called a round. This is rather similar to what happens in astronomy with the helices described daily by the moon owing to its daily motion, which, though not being properly parallel circles, are so called because of this similarity.

4th *Definition*

Those seven steps are called as follows: *ut, re, mi, fa, sol, la, si,* among them the steps from *mi* to *fa* and from *si* to *ut* are [3]) minor steps and all the others major steps.

5th *Definition*

Two sounds having the same pitch, their relation is called selftone. But differing by a minor step, semitone. Differing by a major step, a whole tone. Differing by one major and one minor step, one-tone-and-half. Differing by two major steps, ditone. And so forth in an orderly way.

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[1]) The selected pages which follow have been taken from the part that is in Stevin's own handwriting, with the exception of the chapter on the modes.

[2]) The second definition corresponds to the scale, sung to the vocables *ut, re, mi, fa, sol, la, si, ut, re,* etc.

[3]) The fourth definition was in contradiction to the second definition, in that the step *la* : *si* was here called a minor step, which does not fit in with the sequence major - major - minor - major - major - major - minor - major - etc. We therefore put: *si* to *ut*. If *si* is flattened by a semitone, the resulting note is called *sa*. Thus *la* to *sa* and *si* to *ut* are semitones; *la* to *si* and *sa* to *ut* whole tones.

6e *Bepaling*

Twee gheluyden even hooch sijnde haer verlycking wert oock eerste ghenoemt maer een trap verschillende tweede welcke trap cleyner sijnde heet eyghentlick cleene tweede groot wesende groote tweede: Ghelijcx twee trappen verschillende wort derde gheheijten welcke een cleene sijnde heet cleene derde maer van twee grooter, groote derde endesoovoort tot de sevende wiens volghende trappen dobbeleerst dobbeltweede heeten ende soo oirdentlick voort met de eenvoudiche eerste tweede ende haer volghende.

Verclaring

De singhelicke gheluijden ontfanghen twee verscheyden manieren van namen ghelyckse inde voorgaende 5ᵉ ende 6ᵉ bepalinghen beschreven sijn die elck haer besonder ghebruijck hebben. Want wesende de redens der gheluijden te vergaren ofte van malcander te trecken men noemtse bequamelicker duer de namen der toonen overmidts dat totten tweethoon vergaert den drietoon haer somme is den vyftoon; treckende den tweetoon vanden drie enhalftoon blyft de onderhalftoon inder voughen dat sommen en resten namen der ghetalen cryghen lijckformich an heur sij sijn. De namen van eersten tweeden derden ens. sijn bequamer om int dadelick *) maecksel des sancx te ghebruycken. Want lichter ende bequamelicker telt men tverschil van twee gheluyden deur trappen na de nateurlicke ghesanck climmende of dalende dan deur toonen en halftoonen overmits de menichte der trappen met de menichte der toonen niet en overcomt.

Begheerte

Wij begheeren toeghelaten te werden dat ghelijck snaersdeel tot snaersdeel also haerder gheluyden grofheyt tot grofheyt.

Verclaring

Wanneer twee persoonen tsamen een dobbel eerste singhen de grover stem des leegsten heeft een ghelaet van dobbelheijt teghen de fine stem des hoochsten: dat is ghelijck 2 ellen dobbel sijn teghen 1 elle alsoo schijnt dese leegste stem in grofheijt dobbel te wesen ande hoochste: Tis wel waer dat de selve dobbelheijt ons int gheluijt niet soo heel claer ende verstaenlick en ontmoet als in grootheijt ghetal ghewicht tijt roersel, ende meer ander: nochtans soo beweeght ons de ghespannen snaer selve toe te laten overmits haer deelen in dobbel reden der grootheijt sijnde de selve gheluyden clijncken die wij segghen van dobbel reden der grofheijt te wesen. Want de heel snaer teghen haer helft clincken tsamen de voorschreven dobbel eerste. Voort ghelyck hier gheseyt is, dat de heele snaer tot haer helft in dobbel reden der grootheijt sijnde, haer gheluyt in dobbel reden der grofheijt heeft, alsoo is oock te verstaen dat de heele snaer tot haer vierendeel in verhoudinghe reden der grootheijt wesende haer gheluijt in verhoudinghe reden der grofheyt heeft ende alsoo voort met allen anderen soo wel deelen teghen malcander als deelen teghen de heele snaer.

Nu alsoo ymant mocht willen ontkennen den helft der snaer teghen de heele een dobbel eerste te clijncken, daer uyt oock niet toestaende der gheluyden grofheijt te wesen inde reden van haer snaersdeelen soo wort hier boven beschreven welcke toeghelaten te worden overmidts sulcx als beghinsel gheen ander bewijs en verreyscht. Want ghelijck de ervaring leert, soo en ghebeurt de contrari niet dan deur valsche snaren oft ander ongheval.

*) *Compositione cantus.*

6th *Definition*

Two sounds having the same pitch, their relation is also called a first (or prime), but when differing by one step, it is called a second, and when this step is a minor step, it is properly called a minor second, when it is a major step, a major second. Likewise, when the sounds differ by two steps, their relation is called a third, and when one of them is a minor step, it is called a minor third, but when both are major steps, it is called a major third, and so on to the seventh, the steps following it being called double-first, double-second, and so on in a regular way like the simple first, second, and the following.

Explanation

Singable sounds receive names of two different kinds, as described in the foregoing 5th and 6th definitions, each of which has its special use. For when the ratios of sounds have to be added or subtracted, they are more conveniently referred to by the names of the tones, since, the tritone added to a ditone, their sum is a five-tone; the ditone being substracted from the three-tone-and-half, the remainder is the one-tone-and-half, in such a way that sums and remainders receive names in conformity with the numbers to which they correspond.

The names of firsts, seconds, thirds, etc. are more convenient for practical use in the composition *) of songs. For it is easier and more convenient to count the difference between two sounds by steps ascending or descending according to natural singing than by whole tones and semitones, since the number of the steps does not agree with the number of tones.

Postulate

We postulate that as one part of a string is to another, so is the coarseness of the sound of the one to that of the other.

Explanation

When two persons sing together a double-first, the coarser voice of the lower has an appearance of doubleness with respect to the sharp voice of the higher, i.e. as 2 yards are double to 1 yard, so this lower voice in coarseness seems to be double to the higher. It is true that this doubleness does not present itself quite so clearly and intelligibly in sound as it does in size, number, weight, time, motion, and otherwise; yet the stretched string itself induces us to grant this, since if its parts are in the double ratio as to size, the same sounds ring that we say to be in the double ratio as to coarseness. For the whole string, when played against its half, together make us hear the aforesaid double-first. Further, as it has here been said that, the whole string being to its half in double ratio as to size, its sound is in double ratio as to coarseness, so also it is to be understood that, the whole string being to its quarter in a certain ratio as to size, its sound has the same ratio as to coarseness, and so on for all other cases, parts of the string against each other as well as parts against the whole string.

Now if anyone should wish to deny one half of the string to sound against the whole string as a double-first, and on this account should not admit the coarseness of sounds to be in the ratio of the parts of the string, what is described above has been put as a postulate, since such a postulate in principle does not require any proof. Anyhow, as experience teaches, the contrary does not happen unless owing to false strings or some other mishap.

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* *Compositione cantus.*

2e *Begheerte*

Heele toonen al even groot te wesen, dat sgelijcx oock halve toonen al even groot sijn.

Verclaring

De sin is dese datmen van *ut* tot *re* even soo veel ryst als van *re* tot *mij* ende als van *fa* tot *sol*, van *sol* tot *la* ende van *sa* tot *ut*. Datmen desgelijcx eerst van *mi* tot *fa* even soo hooch rijst als van *la* tot *sa*.

Vertooch (Ghelyck 1 tot)	Alsoo den eenen toon tot den anderen	*Voorstel*
1	Selftoon	Eerste
$\sqrt[(12)]{1/2}$	Halftoon	Cleen tweede
$\sqrt[(6)]{1/2}$	Toon	Groote tweede
$\sqrt[(4)]{1/2}$	Onderhalftoon	Cleen derde
$\sqrt[(3)]{1/2}$	Tweetoon	Groote derde
$\sqrt[(12)]{1/32}$	Twee en halftoon	Vierde
$\sqrt{1/2}$	Drietoon	Qua groote vierde of qua cleene vijfde
$\sqrt[(12)]{1/128}$	Drie en halftoon	Vijfde
$\sqrt[(3)]{1/4}$	Viertoon	Cleen seste
$\sqrt[(4)]{1/8}$	Vier en halftoon	Groote seste
$\sqrt[(6)]{1/32}$	Vijftoon	Cleen sevende
$\sqrt[(12)]{1/2048}$	Vijf en halftoon	Groote sevende
1/2	Sestoon	Dobbeleerste.

Bewijs

[abest]

Vande Reden int ghemeen

Want de redens inde stof des gheluijdts niet soo opentlick bekent en sijn als in ander stoffen daer sij ons ontmoeten, sullen om meerder claerheyt eerst segghen vande Redens ende Everedenheyt int ghemeen; daer nae vande ghedaente des redens der Singconst duer haer verlijcking met de bekende reden der meetconst. Ende ten laetsten van d'eijghen redens der singhelicke gheluyden.

Reden dan int ghemeen bepaelt, is tselver stoffen verlyckingh na de menichvuldenheyt. Als in ghetalen grootheyt, ghewichten, tijt; 6, 6 voeten, 6 pont, 6 uijren, sijn in dobbel reden tot 3, 3 voeten, 3 pont, 3 uijren. D'Everedenheyt is de verlyckinge van twee even redens als 6 tot 3 is een dobbel reden, alsoo oock is 8 tot 4, daerom de reden van 6 tot 3 is even ander reden van 8 tot 4. tsijn dan even redens ende haer verlycking segghende ghelyck 6 tot 3 alsoo 8 tot 4, is everedenheijt ofte 6, 3, 8, 4, sijn everednighe palen.

Siet hier duytsche woorden licht om verstaen ende van slecht ghelaet maer eyghentlick van oneindelick vermueghen. Want soomen ansiet het bepaelde te weten Everedenheijt tis als bepaling sijns grondts, wiens gheluijt alleen, int eerste anhooren ons vermaent ende anwijst dattet recht grontlick verstandt der Evere-

2nd *Postulate*

All whole tones to be equal and likewise all semitones to be equal.

Explanation

The meaning is this: that one rises as much from *ut* to *re* as from *re* to *mi*, and also from *fa* to *sol*, from *sol* to *la*, and from *sa* [1]) to *ut*. That likewise one rises as much from *mi* to *fa* as from *la* to *sa*.

Theorem. [2]) As 1 is to	So one tone to the other	*Proposition*
1	Selftone	First (prime)
$\sqrt{}^{(12)}\, 1/2$	Semitone	Minor second
$\sqrt{}^{(6)}\, 1/2$	Whole tone	Major second
$\sqrt{}^{(4)}\, 1/2$	One-tone-and-half	Minor third
$\sqrt{}^{(3)}\, 1/2$	Ditone	Major third
$\sqrt{}^{(12)}\, 1/32$	Two-tone-and-half	Fourth
$\sqrt{}\, 1/2$	Tritone	Bad major fourth or bad minor fifth
$\sqrt{}^{(12)}\, 1/128$	Three-tone-and-half	Fifth
$\sqrt{}^{(3)}\, 1/4$	Four-tone	Minor sixth
$\sqrt{}^{(4)}\, 1/8$	Four-tone-and-half	Major sixth
$\sqrt{}^{(6)}\, 1/32$	Five-tone	Minor seventh
$\sqrt{}^{(12)}\, 1/2048$	Five-tone-and-half	Major seventh
$1/2$	Six-tone	Double-first (octave)

Proof

(abest)

On Ratio in General

Because ratios in the field of sound are not as manifestly known as in other fields where we meet with them, for the sake of greater clarity we shall first speak about ratios and equirationality [3]) in general; subsequently about the aspect of ratio in singing by comparison with the familiar ratio in geometry. And finally of the ratios proper to musical sounds.

Ratio as defined generally is the relation according to quantity between things of the same nature. As in number, size, weight, time: 6, 6 feet, 6 pounds, 6 hours are in the double ratio to 3, 3 feet, 3 pounds, 3 hours. Equirationality is the relation of two equal ratios. For instance, 6 to 3 is a double ratio, so also is 8 to 4, therefore the ratio of 6 to 3 is equal to the ratio of 8 to 4. They are thus equal ratios, and their relation, *i.e.* saying that as 6 is to 3, so is 8 to 4, is an equirationality, or 6, 3, 8, 4 are terms of an equirationality.

Look here: Dutch words, easy to understand and modest in appearance, but in reality of an infinite power. For if one conciders the thing defined, *viz.* equirationality, it is like a definition of its substance, the mere ring of which, at first hearing, brings home to us and shows us that the very thorough understanding of equira-

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[1]) In the manuscript, erroneously, *si*. This same error has been pointed out in note 3 p. 423. Sometimes Stevin writes the scale as *fa sol la sa ut re mi fa*. Cf. figure 1, p. 436.

[2]) The notation $\sqrt{}^{(n)}a$ is currently used bij Stevin for $\sqrt[n]{a}$. (See volume II B of this edition, p. 524).

[3]) In Stevin's Dutch: *everedenheijt*.

denheyt byde Griecken ende hun navolghers niet gheweest en heeft. Want (veel ander ghelaten die elders te pas sullen comen) te segghen dat 6, 4, 3 van drie ghelycke singconstighe everedenheyt maken daer oneindelicke ydelheden uijt volghen ende besloten worden; Men antwoort duer beweghing van tvoornoemde gheluijt, hier van sijn gheen even redens, daerom oock gheen Everedenheyt. Doirsaeck dier dwalinghen is dat hun spraeck dit woort medtsgaders al d'ander Wisconstighe namen niet soo eyghentlick beteeckenen en conden als dese daerom soomen met goet onderscheyt van der talen nutbaerheijt wilde spreken; men mocht segghen de wetenschap van Griecx oirboir te wesen van veel verscheijden vonden der Griecken die thaerder tyt de voornaemste waren int licht te brenghen, duer oversetting uyt het Griecx in ander talen: sgelijckx daeghelicx ghebeurt; Tlatijn om daer mede (als bij ghevalle des werrelts ghemeen tael gheworden synde) in alle landen verstaen te worden, oock om alle konsten te mueghen besien, die van alle stoffen bij verscheijden geslachten van volcken daerin beschreven worden; Tfranszois Italiaens Spaens Pools, etz om sijn handel daer deur te dryven yder nae sijn ghelegenheyt. Maer het DUYTSCH om de vrie consten daer in te leeren, om de natuerens verborghentheden daer in duergronden ende te bewysen dat wonder gheen wonder en is. Daerom hij die van meyningh waer na de groote Wysheyt te trachten daer der Caldeen ende Egyptenaeren wetenschappen eertijts overblijfselen af waeren, hem soude nut sijn tot desen born oft eerste oirspronck te gaen van daer sijse gekreghen hadden vlietelick in Duytsch leerende onder anderen wat de voornoemde Everedenheyt is. Want dit gheluyt beeldet wesen van dese groote saeck eyghentlick uijt andre woorden als *Proportio Analogia,* synder onbequaem toe ghelyck de daet tot verscheyden plaatsen claerlick betuycht.

Verlijcking der Meetconstighe Reden met de Sinconstighe

Tot hier toe is vande Redens int ghemeen gheseyt maer om nu nae tvoornemen duer verlijcking der meetconstighe Reden die der Singconst te verclaren soo is te weten dat ghelyck de Meetconstighe Reden bestaet in der formen grootheyt ende cleenheyt welcke afghemeten wort duer langhde, alsoo de Singconstens Reden in der gheluyden grofheyt en fynheyt, die afghemeten wort duer hoochde of leechde: Als twee singhende een dobbeleerste, men seght uyt sulck verschil der leechde die deen onder dander is de grofste stem dobbel onder finste. Ende sulcke stof der dobbelheyt als dit is vande selve sijn al d'ander meerder ende minder singconstighe Redens. Wederom ghelyckmen alle Redens van twee voorghestelde rechtlinighe platten of lichamen duer tghesicht niet bekennen en can, maer hun meetconstighe reghels hebben, leerende hoemen die vinden sal, alsoo en sijn alle Redens van twee voorghestelde gheluyden uyt het ghehoor niet te oirdeelen, maer sij worden openbaer duer haer Sinconstighe reghels daer wij nu af segghen moeten.

tionality was not found among the Greeks and their successors. For (leaving aside many other things, to be discussed elsewhere) from saying that 6, 4, 3 of three sounds make a musical equirationality endless vanities follow and are concluded. [1]) The answer, called forth by such a saying is: these form no equal ratios, and consequently there is no equirationality either. The cause of these errors is that the language of the Greeks could not interpret this term together with all other mathematical terms as properly as the Dutch language does. Therefore, if one wished to speak with good discrimination of the suitability of languages, one might say that knowledge of Greek is useful in order to bring to light various discoveries of the Greek, which in their days were the most important, by translation from the Greek into other languages; as is done daily. Latin (because it has happened to become the common language of the world) serves to be understood in all countries, and also to study all sciences about all subjects, which have been described in it by various kinds of people. French, Italian, Spanish, Polish, etc. serve to carry on trade, everybody according to his situation. But DUTCH serves to teach the liberal arts, to fathom the hidden secrets of nature, and to prove that miracle is no miracle.[2]) Therefore, whoso should be minded to seek after the great Wisdom, of which the knowledge of the Chaldees and the Egyptians formerly was a remnant, would find it useful to go to this source or first origin from which they had got it, learning diligently in Dutch, among other things, what is the aforesaid equirationality. For this word depicts the character of this great matter properly. Other words, such as ***Proportio, Analogia,*** are unable to do so, as is clearly shown by practice in several places.

Comparison of Geometrical Ratio with Musical Ratio

So far we have spoken of ratios in general, but in order to explain now, as intended, ratio in singing by comparison with geometrical ratio, we are to know that, as the geometrical ratio consists in the largeness and smallness of figures, which is measured by length, so ratio in singing consists in the coarseness and sharpness of sounds, which is measured by height or lowness. Thus when two persons sing a double-first, it is said, in view of this difference in lowness of one below the other, that the coarser voice is double below the sharper one. And all the other greater or smaller singing ratios are made of the same stuff as the stuff this doubleness is made of. Again, just as all the ratios of two given rectilinear plane figures or solids cannot be recognized by sight, but obey geometrical rules teaching us how to find them, so all the ratios of two given sounds cannot be judged by hearing, but they are revealed by means of the musical rules governing them, which we now have to discuss.

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[1]) Between two numbers p and q one can have an arithmetic mean (a), a harmonic mean (h), and a geometric mean (g). These are defined by $p - a = a - q$; $1/p - 1/h = 1/h - 1/q$, and $p : g = g : q$. Obviously the numbers 6, 4, and 3 quoted by Stevin show the harmonic mean 4 of the outer terms 6 and 3, 4 being $1/3$ more than 3 and $1/3$ less than 6 so that $1/3 - 1/4 = 1/4 - 1/6$. Accordingly the note, given by a length of string 4, is the harmonic mean of the notes given by the lengths 6 and 3. The latter make an octave. The harmonic mean gives a fifth against the lower, a fourth against the higher note. Stevin has in mind geometrical ratio only, and he objects to equating two musical (*singconstighe*) ratios to be construed from the three numbers in question. Obviously he refuses to admit the harmonic ratio to be called a ratio.

[2]) "*Wonder en is gheen wonder*" was the motto on the frontispiece of Stevin's treatise on statics, *De beghinselen der Weeghconst;* see this edition, vol. I, p. 47.

Vande Redens der singchelicke gheluyden
na der Griecken meining

D'ervaring betuycht dat de gespannen snaer op eenich reetschap als luyt cyter viool of derghelycke teghen haer helft een gheluyt maeckt daer mede soo seer ghelyck dattet in hem een ghelaet van selfheyt heeft diens Reden der grofheyt wij duer eenighe natuerlicke gheneghentheyt dobbel verstaen maer niet soo wesentlick als de dobbelheyt die ons in ander stoffen ontmoet, ghelyck vooren gheseyt is, doch soo wort sulcx merckelicker bevesticht duer de lichamen dese gheluijden uijtende als der heelsnaer ende haer helft, welcke oock in dobbel reden sijn. Tselve heeft hem alsoo met de halfsnaer tot huer vierendeel, achtendeel, sesttiendendeel ende d'ander in die voortganck. Want alsulcke gheluyden al tvoornoemde ghelaet der selfheyt hebben, met begrijpelicke ghedaente der viervoudighe achtvoudighe, sesthienvoudighe Reden der grofheijt. Desgelycs is oock openbaer in al dander redens buyten den boveschreven voortganck. Want nemende een deel des snaers wiens Reden tot de heele den helft sij des Redens vande dobbelden haer gheluyt sal oock tot halfweghe duer oirdeelick leeghde ghedaelt sijn: Maer want dese bekende daling de maet der grofheyt is ghelyck wij vooren gheseyt hebben, soo is ons de Reden der grofheijt hier bekent, ende alsoo met anderen dier ghelycken waer uijt besloten wordt dat ghelyck dit snaersdeel tot dat snaersdeel, alsoo desens gheluyt tot diesens gheluyt, dat is de snaersdeelen brenghen gheluyden voort inde Reden haerder grootheden. Dit eertijts bemerckt sijnde, soo was de drangh na de ware deeling des snaers alsoo datse de eijghentlicke toonen begrepen die wij duer natuerlick ghesanck synghen. twelck de Griecken tot onderscheyt van tgheen sy *Chromaticum* ende *Harmonicum* heeten, *Diatonicum genus* noemen, op dat alsoo tnatuerlick ghesanck inde singconstighe reetschappen volcomelick ghetroffen wierden. Om hier toe te commen soo en behouftmen maer eenich toon den halftoon vervatende als onderhalftoon, tweeenhalftoon, drieenhalftoon enz. wantmen daer uijt om der Redens vergaring ende aftrecking wil, al de rest gewislick vinden can sonder meer gheluyden te hooren. Sij hebben daer toe ghenomen de vyfde, dat is den drieenhalftoon ende vinden de ware Reden der langde des snaers ende haers deels desen drieenhalftoon clijnckende seer naer in de Reden van 3 tot 2 hebben gheschat Reden $^3/_2$ de warachtiche te wesen. daermede voortgaende als of syt waer treckense van Reden $^2/_1$ des sestoons blyft Reden $^4/_3$ des tweeenhalftoons, de selve van Reden $^3/_2$ des drieenhalftoons blijft reden $^9/_8$ voor den toon, daer toe vergaert noch een reden $^9/_8$ comt Reden $^{81}/_{64}$ des tweetoons de selve getrocken van Reden $^4/_3$ des tweeenhalftoons blyft voor den halftoon Reden $^{256}/_{243}$, etz. Maer alsmen de sanglijn ofte om werckelicker te spreken, den hals van een luyt of cyter deelt na de boveschreven Redens d'ervaring betuycht opentlick duer tghehoir sulcx den halftoon niet te wesen want sij veel te cleen is. Indervoughe dat de natuerlicke toonen duer sulcke deeling niet recht ghetroffen en sijn. Ende hoewel d'ouden dit ghenomen merckten hebben nochtans dese deeling voor goedt ende volmaeckt ghehouden ende liever tghebreck

On the Ratios of Singable Sounds According to the Opinion of the Greeks

Experience shows that a stretched string on some instrument, such as a lute, a cither, a violin, or the like, produces against its half a sound so similar to it that it has a semblance of identity, the coarseness ratio of which, by some natural inclination, we understand to be double, but not with the same evidence as the doubleness with which we meet in other matters, as has been said before; but this is confirmed more perceptibly by the bodies producing these sounds, such as the whole string and its half, which are also in the double ratio. The same also applies to half the string and its quarter, its eighth, its sixteenth part, etc. in this sequence, for all these sounds have the aforesaid semblance of identity, with the understandable form of the fourfold, eightfold, sixteenfold ratio of coarseness. The same is also obvious in all the ratios besides the above-mentioned sequence. For if we take a part of the string whose ratio to the whole string is one half of the ratio of the double part,[1]) its sound will also have dropped halfway by properly estimated lowness. But because this known dropping is the measure of the coarseness, as we have stated above, here the ratio of coarseness is known to us, and the same also applies to other similar sounds, from which it is concluded that as this part of the string is to that part of the string, so also is the sound of this to the sound of that, *i.e.* the parts of the string produce sounds in the ratio of their sizes.

This being noted in former times, the impulse to the true division of the string was such that it should comprise the notes proper which we sing in natural singing. Which the Greeks, to distinguish it from what they called *Chromaticum* and *Enharmonicum genus*,[2]) called *Diatonicum genus*, in order that thus natural singing might be hit off flawlessly on musical instruments.

To effect this, one merely requires some interval[3]) containing a semitone, such as a one-tone-and-half, a two-tone-and-half, a three-tone-and-half, etc., because from these, by way of addition and subtraction of the ratios, one can accurately find all the rest without hearing any further sounds. For this purpose they took the fifth, *i.e.* the three-tone-and-half, and found the true ratio of the length of the string and its part producing this three-tone-and-half to be very close to the ratio of 3 to 2; they estimated that the ratio 3 : 2 was the true one. Proceeding therewith as if it were the true ratio, they subtracted it from the ratio 2 : 1 of the six-tone, the remainder being the ratio 4 : 3 of the two-tone-and-half; the latter being subtracted from the ratio 3 : 2 of the three-tone-and-half, the remainder is the ratio 9 : 8 for the whole tone. When another ratio 9 : 8 is added thereto, this gives the ratio 81 : 64 of the ditone. When this is subtracted from the ratio 4 : 3 of the two-tone-and-half, the remainder for the semitone is 256 : 243, etc. But if the melodic line, or to speak more concretely: the neck of a lute or cither, is divided according to the above-mentioned ratios, experience shows patently by hearing that this is not the semitone, because it is much too small. Therefore the natural notes are not correctly hit off by such a division. And although the Ancients perceived this fact, nevertheless they took this division to be correct and perfect, and preferred to think that the defect was in our singing

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1) Stevin means the square root of ½.
2) The manuscript, erroneously, has *harmonicum*.
3) Stevin writes: *toon*.

(ghelyck oftmen seyde de Son mach lieghen maer tuijrwerck niet) in ons ghesanck gheacht; ja hebben hierom de soete ende lieflicke gheluijden der cleene ende groote derde en sesten, welcke in haer misdeelde sanglijn mishaeghlick clancken voor quaet ghehouden, te meer dat een sinlicheijt van oneyghen ghetalen hun hier toe drang. Maer willende Ptolemeus daer naer dese onvolmaectheijt verbeteren heeft tvoornoemde *genus diatonicum* op een ander wyse ghedeelt makende onderscheijt tusschen groote toon in Reden $^9/_8$ ende cleene toon in Reden $^{10}/_9$ welck verschil inde natuer niet en bestaet wantet openbaer is alle heele toonen evegroot ghesonghen te worden. Deze onghetroffen toonen van Pitagoras en Ptolemeus an Zarlinus niet ghevallende heeft noch een ander deeling ghemaeckt verspreydende seker *comma* (in Ptolemeus deeling overschietende) op deen en dander toon daert hem goet docht maer al tastende. Alle dese dwalinghen syn daer uyt ghesproten dat den aert der everedenheijt niet grontlick ghenomen begrepen en heeft gheweest twelck niet en quam duer ghebreck des verstants want hun naeghelaten daden ghenouch betuyghen datse van d'alder scherpsinnichsten waren die de natuer voortbrenght maer tlooch hun an goede reetschap naemlick de duytsche tael sonder welcke men inde diepsinnichste saecken soo weijnich doen can als een ervaren timmerman sonder goede verstaelde reetschappen sijn ambacht want ghelyck men duer een ongheschickt cromlinighe form de meetconstighe eygenschappen des viercants niet soo duergronden en can als met een eyghen viercante form na den vyften des 4 voorstels wiens gheduerich opsicht gheduerich tghedacht versterckt alsoo en condemen de diepsinnichste natuerens verborghentheden duer dongheschickte (bij Duytsch verleken) Griecksche spraeck niet soo grontlick begrypen als duer dese aldergheschickste ende aldervolmaeckste tael der talen wiens eijghentlicke beteeckening ons tbeteeckende soo claerlick inbeelt dat de saeck self daer duer gheduerich voor ooghen schijnt welcke in dander talen onbegrijpelicke duysterheden blijven soo dervaring onder anderen in dese stof overvloedelick betuijcht. Want Reden $^3/_2$ voor de vyfde te stellen, daer mede na den vyften voortgaende ende eintlick niet wel uijtcommende noch te meijnen dat Reden $^3/_2$ de waerachtighe sij, voorwaer de grontlicke aert der vergaring ende aftrecking vande Redens isster onbekent. Maer op dat wij dit misverstant in d'onverstaen aert der Redens duer verlijcking van verstaenlicken gheluiden ghetalen openbaer maken: laet ons nemen eenich ghetal als 110 inde plaets der dobbeleerste of des sesthoons, ende vyf persoonen, A, B, C, D, E, oirdentlick beteeckenende den drienhalftoon tweeenhalftoon, toon, tweetoon ende halftoon, daer mede den eysch stellende lyckformich ande voorgaende Pitagorische wercking des Redens aldus: *Van 110 ghetrocken tghene A hebben moet de rest is voor B, ende ghetrocken B van A t'overschot is voor C, daer toe noch soo veel ghedaen de somme is voor D, die ghetrocken van B toverblyfsel moet 35 syn voor E.* Ymant om tot besluyt van desen te commen, neemt een ghetal voor A dat hem soo veel uyterlick ghevoel belanght na ghenouch dunckt als 60, hier mede voortgaende als oftet twaerachtich waer, trecket van 110 blyft 50 voor B, die ghetrocken vande (as if one should say: the sun may lie, but the clock cannot). They even considered the sweet and lovely sounds of the minor and the major third and sixth, which sounded unpleasant in their misdivided melodic line, to be wrong, the more so because a dislike for inappropriate numbers moved them to do so. But when Ptolemy afterwards wanted to amend this imperfection, he divided the aforesaid *genus diatonicum* in a different way, making a distinction between a major whole tone in the ratio 9 : 8 and a minor whole tone in the ratio 10 : 9, a difference that does not exist in nature, for it is obvious that all whole tones are sung equal. Since these tones of Pythagoras and of Ptolemy displeased Zarlino, he made yet another division, distributing a certain *comma* (which remained in Ptolemy's division) over one tone and another, where it seemed appropriate to him, but tentatively.[1])

All these misconceptions have originated from the fact that fundamentally the nature of equirationality was not understood, which was not due to a lack of brains, for their acts as have come down to us show sufficiently that the Greek were of the most intelligent that Nature produces, but they lacked a good tool, *viz.* the Dutch language, without which in the most profound matters one can accomplish as little as a skilled carpenter without good tempered tools can carry out his trade. For just as one cannot grasp the geometrical properties of a square by means of an unsuitable curvilinear figure so well as by means of a proper square figure according to Proposition 4, sub 5,[2]) the continual sight of which continually strengthens one's insight, so it was not possible to penetrate into the most profound secrets of Nature as thoroughly by means of the unsuitable Greek language (unsuitable as compared to Dutch) as by means of this eminently suitable and most perfect language of languages, whose characteristic designation pictures the matter designated so clearly for us that the matter itself thus seems to be continually before our eyes, whilst in other languages it remains incomprehensible and obscure, as experience amply proves, among other things in the present matter. For whoso puts the ratio 3 : 2 for the fifth, proceeding therewith five times, and in the end not coming out right, still holds that the ratio 3 : 2 is the actual one, he in truth ignores the essential character of addition and subtraction of ratios.

But in order to show up this misapprehension as to the misunderstood character of ratios by means of an example in intelligible words with numbers, let us take some number, such as 110, as representing the double-first, or the six-tone, and five persons A, B, C, D, E, representing in this order the three-tone-and-half, the two-tone-and-half, the whole tone, the ditone, and the semitone, putting therewith the requirement similar to the preceding Pythagorean operation with the ratio, as follows: *Subtract from* 110 *what A should have, the remainder is for B; subtract B from A, the remainder is for C; add to this the same amount, then the sum is for D; after subtraction of the latter from B, the remainder should be* 35 *for E.* To arrive at this result, somebody takes for A a number which as a superficial guess appears to him close enough, for instance 60. Proceeding with this as if it were the true number, he subtracts it from 110, there remains 50 for B; this

[1]) For this question, see note A, page 460.
[2]) Reference to an item of Stevin's treatise *De Meetdaet,* Work XI, this edition vol. IIB.

60 rest 10 voor C, daer toe ghedaen noch 10 vint 20 voor D, die ghetrocken van 50 der B blyft 30 voor E, maer E moest 35 hebben hij siet dan opentlick dat E tsijne niet en heeft; doch sonder te mercken dat sulcx comt uijt het eerste ghetal voor A dats 60 onrecht ghestelt te wesen acht dat laetste ongheval de naturens verborchenheijt houdende sijn boveschreven besluyt voor goet. Maer wat sal den ervaren Telder hier toe segghen? seker met goede reden dat soodanighen deyghenschappen der Telconst niet ghenough bekent en sijn, wetende dattet recht deel voor A 59 is, twelck van 110 ghetrocken blyft 51 voor B, welcke van 59 rest 8 voor C, daer toe noch 8 comt 16 voor D, die ghetrocken van 51 der B blyft 35 voor E naer tbegheerde. Even eens ist inde berekening vande Redens der gheluyden toegeghaen, want wesende voor de vyfde een Redens te stellen, die na seecker reghel af te trecken ende te vergaren was, alsoo datter eintlick de ware Reden des halftoons overschiete welcke men duert stellen van Reden $^{3}/_{2}$ daetlick bevandt daer niet uijt te commen, ende bevandt noch gheduerlick te blyven meijnen dat die Reden $^{3}/_{2}$ de waerachtighe is; Voorwaer soo opentlick als den Telder hier boven sach dat den stelder van 60 voor A de Telconst niet ghenouch en verstont naestelick ghevoelende doirsaeck sijnder dwaling; even soo claerlick siet den ervaren der Everedenheijt dese stelders van Reden $^{3}/_{2}$ voor de vyfde den gront-licken aert der Redens end Everedenheyts niet innerlick ghenomen begrepen te hebben spruijtende daer uijt als voor gheseyt is dat sij gheen woorden en hadden die de Wisconstighe saken soo eyghentlick beteeckenen conden als het DUYTSCH.

Vande ware redens der natuerlicke toonen

Maer om tot de saeck te commen ende deyghen Redens der natuerlicke toonen te beschryven, soo segh ick dat de ware reden der vyfden ofte des drieenhalftoons is van 1 tot $\sqrt{(12)}\ ^{1}/_{128}$, dat is van 1 tot syde der twelfde grootheyt van $^{1}/_{128}$, de selve ghetrocken van Reden $^{2}/_{1}$ des sestoons blyft Reden van 1 tot $\sqrt{(12)}\ ^{1}/_{32}$ voor den tweeenhalftoon, die wederom ghetrocken vande voornoemde Reden des drieenhalftoons blijft Reden van 1 tot $\sqrt{(6)}\ ^{1}/_{2}$ voor den toon, daer toe ghedaen noch alsulcken reden comt Reden van 1 tot $\sqrt{(3)}\ ^{1}/_{2}$ voor den tweetoon, de selve ghetrocken vande boveschreven Reden des tweeenhalftoons blyft reden van 1 tot $\sqrt{(12)}\ ^{1}/_{2}$ voor den halftoon. Om twelck te bewysen soo laet A, B, C, D, E, F, G, *a, b, c, d, e, f, g,* beteeckenen de clawieren van een orgel ofte clavesingel ende H I K L M N O P Q R de tusschen toonen diese fenten noemen. Want ons dit reetschap tottet voornemen bequamer is, dan de sanglijn, tselve laet ghestelt worden met de volmaeckte natuerlicke toonen in deser voughen

Boven F de dobbeleerste f met de vijfde *c* tusschen beyden
Onder *c* de dobbeleerste C met de vijfde G tusschen beyden
Boven G de dobbeleerste *g* met de vijfde *d* tusschen beyden
Onder *d* de dobbeleerste D met de vijfde *a* tusschen beyden
Onder *a* de dobbeleerste A met de vijfde E tusschen beyden
Boven E de dobbeleerste *e* met de vijfde *b* tusschen beyden
Onder *b* de dobbeleerste B met de vijfde L tusschen beyden
Boven L de dobbeleerste Q met de vijfde O tusschen beyden

being subtracted from 60, there remains 10 for C; adding 10 again to this, he finds 20 for D; this being subtracted from the 50 of B, there remains 30 for E. But E was to have 35, so he sees patently that E has not received his due. But not perceiving that this follows from the fact that the first number for A, *i.e.* 60, had not been put right, he considers the last mishap to be the secret of Nature and looks upon his former supposition as correct. But what will the experienced arithmetician say to this? Certainly, and with good reason, that such a person is insufficiently acquainted with the properties of arithmetic, for he knows that the correct portion for A is 59; when this is subtracted from 110, there remains 51 for B; this from 59, there remains 8 for C; adding another 8, that makes 16 for D; this subtracted from 51 of B, there remains 35 for E, as required.

The same thing happened in the calculation of the ratios of sounds, for a ratio had to be put for the fifth and this had to be subtracted and added according to a given rule, in such a way that finally the true ratio of the semitone should remain. In fact this was found not to come right if the ratio was put to be 3 : 2, and people continually went on maintaining this ratio 3 : 2 to be the true one. Truly, as patently as the arithmetician above saw that the man who put 60 for A did not sufficiently understand arithmetic, feeling at once the cause of his error, so clearly the expert in equirationality sees that the supporters of the ratio 3 : 2 for the fifth have not essentially understood the fundamental character of ratios and equirationalities, which was due, as said above, to the fact that they had no words which could designate mathematical matters as adequately as DUTCH.

Of the True Ratios of Natural Tones

But, to come to the point and to describe the proper ratios of natural intervals[1]), I say that the true ratio of the fifth or the three-tone-and-half is 1 to $\sqrt{(12)}\ {}^1/_{128}$, *i.e.* 1 to the twelfth root of $^1/_{128}$. When this is subtracted from the ratio 2 : 1 of the sixtone, there remains the ratio of 1 to $\sqrt{(12)}\ {}^1/_{32}$ for the two-tone-and-half. When this again is subtracted from the aforesaid ratio of the three-tone-and-half, there remains the ratio of 1 to $\sqrt{(6)}\ {}^1/_2$ for the whole tone. Addition to this of the same ratio makes the ratio of 1 to $\sqrt{(3)}\ {}^1/_2$ for the ditone. This being subtracted from the above-mentioned ratio of the two-tone-and-half, there remains the ratio of 1 to $\sqrt{(12)}\ {}^1/_2$ for the semitone. To prove this, let A, B, C, D, E, F, G, *a, b, c, d, e, f, g* designate the keys of an organ or a harpsichord, and H, I, K, L, M, N, O, P, Q, R the intermediate keys, which are called slit keys. Because this instrument is more convenient for the present purpose than the monochord, let it be tuned with the perfect natural tones, as follows:

Above F the double-first *f* with the fifth *c* between them
Below *c* the double-first C with the fifth G between them
Above G the double-first *g* with the fifth *d* between them
Below *d* the double-first D with the fifth *a* between them
Below *a* the double-first A with the fifth E between them
Above E the double-first *e* with the fifth *b* between them
Below *b* the double-first B with the fifth L between them
Above L the double-first Q with the fifth O between them

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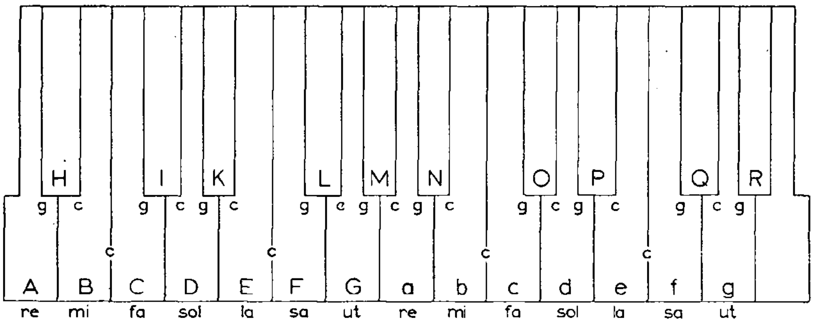
1) Stevin writes: toonen *(= tones)*.

Onder O de dobbeleerste I met de vijfde M tusschen beyden
Boven M de dobbeleerste R met de vijfde P tusschen beyden
Onder P de dobbeleerste K met de vijfde N tusschen beyden
Onder N de dobbeleerste H



Dit soo wesende dervaring betuycht dat HF een volmaeckte vyfde maken, ende hoewel sulcx voor ghemeen ende ghewisse Reghel gehouden wort van al de ghene hun dies verstaende, heb nochtans tot meerder versekering voor de ghene die daer an twijfelen mocht de Loofweerdicheyt willen ghebruijcken van . . .

Wesende dan HF een volmaeckte vijfde, soo syn alle halftoonen nootsaecklick evegroot ende den rechten helft des toons twelck aldus bewesen wort: Laet den halftoon van B tot C ende van E tot F cleynder oft grooter syn, waert mueghelick dan den rechten helft des toons; ick neem na de Pitagorische meining cleender, twelck wij daerom (metgaders *bc* ende *ef*) teyckenen met *c* clein bediende, duer tlettercken *g* salmen groot halftoon verstaen: Om dan voort te gaen LB is duer de stelling een vijfde bestaende uijt drie toonen ende een cleen halftoon, ofte dattet selve is uyt twee toonen, twee cleene halftoonen met een groot halftoon; Dit soo synde van F tot L is een groot halftoon twelck aldus bewesen wort. BC is een cleen halftoon CD ende DE elck een toon, EF een cleen halftoon maken tsamen twee toonen ende twee cleene halftoonen, soo moet dan FL tot voldoening der vyfde BL een groot halftoon sijn, ende vervolghens van L tot G is een cleen halftoon want van F tot G.is een toon daer af ghetrocken de groot halftoon van F tot L soo moet dan L tot G een clein halftoon sijn. Maer *fQg* sijn dobbeleersten met FLG daerom oock ist van *f* tot Q een groot halftoon ende van Q tot

Below O the double-first I with the fifth M between them
Above M the double-first R with the fifth P between them
Below P the double-first K with the fifth N between them
Below N the double-first H

Figure 1. Plan of a harpsichord's keybord. Between the keys, semitones supposed minor have been marked c by Stevin (from cleen = small); major semitones by g (from groot = large). Starting the scale with ut on G, Stevin in f has sa, on a white key.

This being so, experience shows that H and F make a perfect fifth, and although this is considered a common and certain rule by all those who are skilled in this matter, yet to convince those who should doubt it I thought fit to use the authority of . . . . . [1])

Thus from H to F being a perfect fifth, all semitones must needs be equal and an exact half of a whole tone, which is proved as follows.

Let the semitone from B to C and from E to F be smaller or larger, if possible, than a right half tone; I assume, according to the Pythagorean view, that it is smaller, which we therefore (as also *b : c,* and *e : f*) designate by *c* [2]), meaning small; the letter *g* must be taken to stand for a major semitone. To proceed, L to B, by tuning, is a fifth, consisting of three whole tones and a minor semitone or, which is the same, of two whole tones, two minor semitones, and one major semitone. This being so, from F to L is a major semitone, which is proved as follows. BC is a minor semitone, CD and DE each a whole tone, EF a minor semitone, making together two whole tones and two minor semitones; therefore FL, to make up the fifth BL, must be a major semitone, and consequently from L to G is a minor semitone, for from F to G is a whole tone; subtracting from this the major semitone from F to L, from L to G must be a minor semitone. But *f,* Q, *g* make double-firsts with F, L, G, therefore from *f* to Q is a major semitone

[1]) The same diagram, and the method of tuning, in which Stevin uses the expressions *doctaaf* (the octave) and *de quinte* (the fifth) was shown on a separate leaflet. A foot-note also was on a separate slip of paper. In the note Stevin supposes that in the tuning experiment one has started from E-flat (the keys K and P in the diagram). In that case the last step leads to G-sharp (*gis,* the keys M and R). Stevin argued that the people quoted by him proclaim that they find the starting note P to be identical with the perfect fifth (d-sharp or *dis*) above M. Hence, he says, F too is a perfect fifth above H.

[2]) *c,* taken from Dutch *cleen* = small. The letter *r* in earlier publications seems to be corrupt. The letter *g* is taken from Dutch *groot* = large.

g een cleen halftoon. Voort soo is O een vyfde op L. duer de stelling daerom oock ist van *c* tot O een groot halftoon twelck aldus bethoont wort: LG is een cleen halftoon, G*b* twee toonen, *bc* een cleen halftoon, maken tsamen twee toonen ende twee cleene halftoonen soo moet dan *c*O tot voldoening der vijfde LO een groote halftoon sijn, ende vervolghens soo is O*d* een cleen halftoon. Maer CID sijn dobbeleersten met *c*O*d* daerom oock ist van C tot I een groot halftoon ende van I tot D een cleen halftoon. Voort soo is M een vyfde op I duer de stelling, daerom oock ist van G tot M een groot halftoon want ID is een cleen halftoon ende DE een toon, EF een cleen halftoon, FG een toon maken tsamen twee toonen ende twee cleene halftoonen indervoughen dat GM tot voldoening der vyfde MI een groot halftoon maken ende vervolghens soo is M*a* een cleen halftoon. Maer *gR* sijn dobbeleersten met GM, daerom oock is *gR* een groot halftoon.

Wyder soo is P een vyfde op M duer de stelling daerom oock ist van *d* tot P een groot halftoon want van M tot *a* is een cleen halftoon van *a* tot *b* een toon, van *b* tot *c* een cleen halftoon, van *c* tot *d* een toon, maken tsamen twee toonen ende twee cleene halftoonen waer duer *d*P tot voldoening der vyfde PL nootsaeckelick een groot halftoon is, ende vervolghens soo moet P*e* een cleen halftoon sijn. Maer DKE sijn dobbeleersten met *d*P*e* daerom oock is DK een groote halftoon ende KE een cleen halftoon. Voort soo is N den vijfde op K duer de stelling daerom ist oock van *a* tot N een groot halftoon. Want van K tot E is een cleen halftoon ende van E tot F oock een cleen halftoon ende van F tot *a* twee toonen, maken tsamen twee toonen ende twee cleene halftoonen waer duer *a*N tot voldoening der vyfde NK een groot halftoon maeckt, ende vervolghens N*b* een cleen halftoon, maer AHB sijn dobbeleersten inde *a*N*b*, daerom oock is AH een groote halftoon ende HB een cleen halftoon. Dit dus wesende HF bestaet uyt twee toonen ende drie cleyn halftoonen. Want HB is een cleen halftoon, alsoo oock is BC, ende CE sijn twee toonen ende EF een cleen halftoon maken tsamen als vooren gheseijt is, twee toonen ende drie cleene halftoonen. HF dan en is gheen vyfde twelck teghen dervaring teghen loofweerdicheyt teghen tghemeen ghevoelen ende ontkenning der beginselen soude sijn; merckt wijder dat soo veel BC cleender waer dan een recht halftoon soo veel soude AH nootsaeckelick grooter moeten wesen ende vervolghens haer verschil tot malcander tweemael soo veel twelck teghen tghemeen ghevoelen is. Want ghelyck int ghesanck de climming van *mi* tot *fa* evensoo hooch is als van *la* tot *sa*, alsoo istter van B tot C evensoo veel rysing als van A tot H. BC dan en is niet minder dan den rechten helft eens toons; sghelycx salmense oock bewysen niet meerder te wesen, sij is dan nootsakelick den rechten helft, alsoo oock sijn al dander ghelyck van A tot H, van H tot B, enz. Dit soo wesende de dobbeleerste bestaet nootsaecklick in ses toonen al even groot ofte in twelf evegroote halftoonen daerom heeft men tbegheerde alsder tusschen de palen der dobbeleersten 1 ende $^1/_2$ gheteyckent hier onder met AB ghevonden sijn. elf middeleverednighe ghetalen C, D, E, F, G, H, J, K, L, M, N, aldus and from Q to *g* a minor semitone. Further, O is a fifth above L, by tuning, therefore from *c* to O is a major semitone, which is proved as follows: LG is a minor semitone, G*b* two whole tones, *bc* a minor semitone, making together two whole tones and two minor semitones. Therefore *c*O, to make up the fifth LO, must be a major semitone, and consequently O*d* is a minor semitone. But *c*, O, *d* make double-firsts with C, I, D, therefore from C to I is a major semitone and from I to D a minor semitone. Again, M is a fifth above I, by tuning; therefore from G to M is a major semitone, for ID is a minor semitone and DE a whole tone, EF a minor semitone, FG a whole tone, making together two whole tones and two minor semitones, so that GM, to make up the fifth IM, is a major semitone, and consequently M*a* is a minor semitone. But *g*, R make double-firsts with G, M, therefore *g*R is a major semitone.

Further, P is a fifth above M, by tuning, therefore from *d* to P is a major semitone also, for from M to *a* is a minor semitone, from *a* to *b* a whole tone, from *b* to *c* a minor semitone, from *c* to *d* a whole tone, making together two whole tones and two minor semitones, in consequence of which *d*P, to make up the fifth LP, must needs be a major semitone, and consequently P*e* must be a minor semitone.

But *d*, P, *e* make double-firsts with D, K, E; therefore DK too is a major semitone and KE a minor semitone. Further N is a fifth above K, by tuning, therefore from *a* to N is a major semitone. Because from K to E is a minor semitone and from E to F also a minor semitone, and from F to *a* two whole tones, this makes together two whole tones and two minor semitones, in consequense of which *a*N, to make up the fifth KN, makes a major semitone, and consequently N*b* is a minor semitone; but *a*, N, *b* make double-firsts with A, H, B, therefore AH is a major semitone and HB a minor semitone. This being so, HF consists of two whole tones and three minor semitones. For HB is a minor semitone; such is BC too, and CE are two whole tones and EF is a minor semitone; these make together, as said above, two whole tones and three minor semitones. HF therefore is no fifth, which would be contrary to experience, contrary to authority, contrary to common opinion, and a denial of the principles. Note further that so much smaller as BC would be than a right semitone, so much larger AH must needs be, and consequently the difference between them twice as much, which is contrary to the common opinion. For just as in singing the ascent from *mi* to *fa* is equal to that from *la* to *sa*, so the ascent from B to C is equal to that from A to H. BC therefore is not less than the right half of a whole tone; likewise one will also show it not to be more. Therefore it must needs be the right half, and thus all the others, from A to H, from H to B, etc., are also equal. This being so, the double-first must needs consist of six whole tones which are all equal, or of twelve equal semitones; therefore the requirement will be satistied if, between the bounds of the double-first 1 and $^1/_2$, designated below by A and B, there have been found eleven mean proportional numbers C, D, E, F, G, H, I, K, L, M, N [1]), as follows:

[1]) Mean proportionals, forming a geometric progression.

A. 1	Selftoon	Eerste
C. $\sqrt{(12)}\,1/2$	Halftoon	Cleen tweede
D. $\sqrt{(6)}\,1/2$	Toon	Groote tweede
E. $\sqrt{(4)}\,1/2$	Onderhalftoon	Cleen derde
F. $\sqrt{(3)}\,1/2$	Tweetoon	Groote derde
G. $\sqrt{(12)}\,1/32$	Tweeenhalftoon	Vierde
H. $\sqrt{}\,1/2$	Drietoon	Qua groote vierde of qua cleene vijfde
I. $\sqrt{(12)}\,1/128$	Drieenhalftoon	Vijfde
K. $\sqrt{(3)}\,1/4$	Viertoon	Cleen seste
L. $\sqrt{(4)}\,1/8$	Vierenhalftoon	Groote seste
M. $\sqrt{(6)}\,1/32$	Vijftoon	Cleen sevende
N. $\sqrt{(12)}\,1/2\,048$	Vijfenhalftoon	Groote sevende
B. $1/2$	Sestoon	Dobbeleerste, achtste.

Inder voughen dat A, 1 tot A, 1 de reden des selftoons ofte der eerste is, maer A, 1 tot C, $\sqrt{(12)}\,1/2$ de reden des halftoons ofte der cleen tweede ende A, 1 tot D, $\sqrt{(6)}\,1/2$ de reden des toons ofte der groote tweede ende soo voort met de rest, waer uijt blyckt dat de vyfde en dander in sulcke redens zijn als wij voorghenomen hadden te bewysen.

Ymant mocht nu achten na doude meyning hoe dattet soet gheluydt der vyfde in soo *) onuijtsprekelick, onredelick, ongeschickt ghetal bestonde, daer op wij int breede souden connen antwoorden maer want ons voornemen niet en is an donuytsprekelicke onredelicheyt ende ongeschicktheyt van sulcken misverstant hier te leeren duytsprekelicheyt redelicheyt geschicktheijt ende natuerlicke constighe volmaecktheyt deser ghetalen, sullent, als elders bewesen hebbende, daer bij laten.

Maer soomen de boveschreven redens al wilde beteeckenen met syden en twelfde grootheden inde selve weerde men soude den voortganck des noemers vande ghebroken in oirdentlicke voortganck vinden waeruyt, duer lichticheyt bekent worden al de redens boven den sestoon ofte dobbeleerste ghelyck dit voorbeelt opentlick ghenouch aenwyst:

*) *Inexplicabili irrationali absurdo numero.*

A. 1	Selftone	First
C. $\sqrt[(12)]{1/2}$ . . .	Semitone	Minor second
D. $\sqrt[(6)]{1/2}$ . . .	Whole tone	Major second
E. $\sqrt[(4)]{1/2}$ . . .	One-tone-and-half	Minor third
F. $\sqrt[(3)]{1/2}$ . . .	Ditone	Major third
G. $\sqrt[(12)]{1/32}$ . . .	Two-tone-and-half	Fourth
H. $\sqrt{1/2}$	Tritone	Bad major fourth or bad minor fifth
I. $\sqrt[(12)]{1/128}$ . .	Three-tone-and-half	Fifth
K. $\sqrt[(3)]{1/4}$ . . .	Four-tone	Minor sixth
L. $\sqrt[(4)]{1/8}$ . . .	Four-tone-and-half	Major sixth
M. $\sqrt[(6)]{1/32}$ . . .	Five-tone	Minor seventh
N. $\sqrt[(12)]{1/2048}$ . .	Five-tone-and-half	Major seventh
B. $1/2$	Six-tone	Double-first, eighth

Thus, A,1 to A,1 is the ratio of the selftone or first, but A,1 to C,$\sqrt[(12)]{1/2}$ the ratio of the semitone or minor second, and A,1 to D,$\sqrt[(6)]{1/2}$ the ratio of the whole tone or the major second, and so on with the remaining ratios, from which it appears that the fifth and the others are in such ratios as we had proposed to prove.

Now, someone might wonder, according to the ancient view, how the sweet sound of the fifth could consist in so *) unspeakable, irrational, and inappropriate a number [1]). To this we might give a detailed answer. However, since it is not our intention to teach to the unspeakable irrationality and inappropriateness of such a misunderstanding the speakability, rationality, appropriateness, and natural wonderful perfection of these numbers, we shall leave it at that because we have proved it elsewhere.

But if one wished to express all the above mentioned ratios by twelfth roots, one would find the progression of the denominators of the fractions in a regular sequence, from which all the ratios above the six-tone or double-first become easily known, as the following example shows sufficiently clearly:

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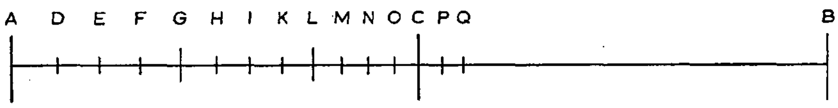
*) *Inexplicabili irrationali absurdo numero.*

[1]) Referring to his work on arithmetic, (Work V, Vol. II B of this edition, p. 532; Vol. I, p. 23), Stevin in advance ridicules objections concerning the irrationality of his numbers.

A tot A	A $\sqrt[(12)]{1}$	Selftoon	Eerste
A » H	C $\sqrt[(12)]{1/2}$	Halftoon	Cleen tweede
A » B	D $\sqrt[(12)]{1/4}$	Toon	Groote tweede
A » C	E $\sqrt[(12)]{1/8}$	Onderhalftoon	Cleen derde
A » I	F $\sqrt[(12)]{1/16}$	Tweetoon	Groote derde
A » D	G $\sqrt[(12)]{1/32}$	Tweeenhalftoon	Vierde
A » K	H $\sqrt[(12)]{1/64}$	Drietoon	Qua groote vierde of qua cleen vyfde
A » E	I $\sqrt[(12)]{1/128}$	Drieenhalftoon	Vyfde
A » F	K $\sqrt[(12)]{1/256}$	Viertoon	Cleen seste
A » L	L $\sqrt[(12)]{1/512}$	Vierenhalftoon	Groote seste
A » G	M $\sqrt[(12)]{1/1\,024}$	Vijftoon	Cleen sevende
A » M	N $\sqrt[(12)]{1/2\,048}$	Vijvenhalftoon	Groote sevende
A » *a*	B $\sqrt[(12)]{1/4\,096}$	Sestoon	Dobbeleerste, achste
	$\sqrt[(12)]{1/8\,192}$	Sessenhalftoon	Dobbel cleen tweede
	$\sqrt[(12)]{1/16\,384}$	Sevetoon	Dobbel groote tweede
	$\sqrt[(12)]{1/32\,768}$	Sevenenhalftoon	Dobbel cleen derde.

*Meetconstighe deeling der *) sanglini*

Om nu de sanglijn meetconstlick te deelen alsoo datmen daer in hebbe de ware volcommen gheluyden des natuerlicken ghesancks dat is inde boveschreven redens so laet AB de sanglijn beteeckenen wiens middelpunt C is, de selve salmen deelen in D, E, F, G, H, I, K, L, M, N, O, alsoo dat GB ende LB. twee middeleverednighe lynen sijn tusschen AB ende CB die op verscheyden wyse, werckelick (want de wisconstighe is alsnoch onbekent) ghevonden wort doch bequamelicst mijns bedunckens na de manier van . . . . Voort alsoo dat IB middeleveredniche sij tusschen AB ende CB, diemen vindt duer het.... Voor-



stel des.... boucs van Euclides. Sghelycx EB tusschen AB en GB. Wederom DB tusschen AB en EB. Voort FB tusschen AB en IB. Sghelycx HB tusschen GB en IB; Ende KB tusschen EB en CB. Wederom MB tusschen IB en CB ende NB tusschen LB en CB. Ten laetsten OB tusschen NB en CB.

Maer soomen dese deelinghen van C naer B noch voorder begheerde dat can met lichticheyt geschien in deser voughen: Men sal teeckenen P int middel van DB ende Q int middel van EB ende soo voort want PB sal teghen AB den sesenhalftoon ofte dobbel cleentweede maken ende QB teghen AB den sevetoon ofte dobbelgroote tweede.

*) *Regula harmonices seu monocordus.*

A to A	A	$\sqrt[(12)]{1}$	Selftone	First [1]
A to H	C	$\sqrt[(12)]{1/2}$	Semitone	Minor second
A to B	D	$\sqrt[(12)]{1/4}$	Whole tone	Major second
A to C	E	$\sqrt[(12)]{1/8}$	One-tone-and-half	Minor third
A to I	F	$\sqrt[(12)]{1/16}$	Ditone	Major third
A to D	G	$\sqrt[(12)]{1/32}$	Two-tone-and-half	Fourth
A to K	H	$\sqrt[(12)]{1/64}$	Tritone	Bad major fourth or bad minor fifth
A to E	I	$\sqrt[(12)]{1/128}$	Three-tone-and-half	Fifth
A to F	K	$\sqrt[(12)]{1/256}$	Four-tone	Minor sixth
A to L	L	$\sqrt[(12)]{1/512}$	Four-tone-and-half	Major sixth
A to G	M	$\sqrt[(12)]{1/1\,024}$	Five-tone	Minor seventh
A to M	N	$\sqrt[(12)]{1/2\,048}$	Five-tone-and-half	Major seventh
A to *a*	B	$\sqrt[(12)]{1/4\,096}$	Six-tone	Double-first eighth (octave)
		$\sqrt[(12)]{1/8\,192}$	Six-tone-and-half	Double minor second
		$\sqrt[(12)]{1/16\,384}$	Seven-tone	Double major second
		$\sqrt[(12)]{1/32\,768}$	Seven-tone-and-half	Double minor third

*Geometrical Division of the * Monochord*

Now in order to divide the monochord geometrically in such a way that we have the true perfect sounds of natural singing, *i.e.* in the above mentioned ratios, let AB designate the monochord, whose centre is C. Divide this in D, E, F, G, H, I, K, L, M, N, O in such a way that GB and LB are two mean proportional lines between AB and CB [2]), which may be found in practice (for the mathematical method is still unknown) by different methods, but in my opinion most conveniently in the manner of . . . .

Figure 2. Division of the monochord in equal tones and semitones.

Further in such a way that IB be the mean proportional between AB and CB, which is found by the . . th Proposition of the . . th Book of Euclid. Likewise EB between AB and GB. Again DB between AB and EB. Further FB between AB and IB. Likewise HB between GB and IB; and KB between EB and CB. Again MB between IB and CB, and NB between LB and CB. Finally OB between NB and CB.

But if these divisions from C to B are required to be continued, this can easily be done as follows. Mark P in the middle of DB and Q in the middle of EB, and so on, because PB will make against AB the six-tone-and-half or double minor second, and QB against AB the seven-tone or double major second.

1) The letters in the first column of the following table correspond to the diagram, fig. I, p. 436. In the second column they refer to the preceding table on p. 441.

2) AB : GB = GB : LB = LB : CB.

*) *Regula harmonices seu monocordus.*

Telconstighe deeling der sanglijn

Tot hier toe is vande Meetconstighe deeling gheseyt; int volghende sullen wij de telconstighe verclaren dat is duer slechte ghetalen inde daet ghenouch doende, aldus: *Ic deel de lijn AB in 10 000 even deelen; nu om te weten hoeveel der* selver tot yder toon behooren ist beghin anden drieenhalftoon segghende 1 gheeft $\sqrt{}^{(12)}\, {}^{1}/_{128}$ wat 10 000? comt
$\sqrt{}^{(12)}$7 812 500 000 000 000 000 000 000 000 000 000 000 000 000 000
die doet zeer bij ende in heel ghetal ten naesten 6 674. Want 6 675 te veel is soo duer de twelfde grootheyt van deen en dander blijcken can. Maer om eyghentlicke bewys te doen in yder deel der wercking vande vinding deser syden soo is te weten dat de syde der tweede grootheyt ofte viercants syde des boveschreven vierde paels is bina $\sqrt{}^{(6)}$88 388 347 648 318 440 550 105 (die de gront hier af begheert mach dit ghetal in sich self menichvuldighen daer bij doende 96 389 809 968 824 984 488 975 dieder overschoten; tblyckt oock duer tvoornomde overschot dat de ware syde van gheen eenheyt meerder en is maer alleenlick van ontrent $\frac{96\ 389\ 809\ 968\ 824\ 984\ 488\ 975}{176\ 776\ 695\ 296\ 636\ 881\ 100\ 211}$).
Vande voornomde $\sqrt{}^{(6)}$ 88 388 347 648 318 440 550 105 wederom ghetrocken viercants syde die doet $\sqrt{}^{(3)}$ 297 301 778 750 (de prouf is openbaer duer menichvuldighing deses ghetals in sich daer toe doende doverschietende 404 488 987 605, tblyckt oock duer tvoornomde overschot dat de ware syde van gheen eenheyt meerder en is maer alleenlick van ontrent $\frac{404\ 488\ 987\ 605}{594\ 603\ 557\ 501}$ *). Hier uyt ten laetsten ghetrocken syde der derde grootheyt ofte teerlinxsyde comt 6 674, de prouf van desen is dat 6 674 teerlincwijs ghemenichvuldicht ende daer toe ghedaen doverschietende 26 628 726 maken haer eerste teerlincstal, tblyckt oock duer tvoornomde overschot datde ware teerlincxsyde van gheen eenheyt meerder en is maer alleenlick van ontrent $\frac{26\ 628\ 726}{133\ 646\ 851}$. Inder voughen dat 6 674 de voornomde syde in heeltal ten naesten is. De Reden die des drieenhalftoons is in slechttal te nemen als van 10 000 tot 6 674, daervan ghetrocken Reden ${}^{10\,000}/_{6\,674}$ des drieenhalftoons van Reden ${}^{2}/_{1}$ des sestoons blyft Reden ${}^{13\,348}/_{10\,000}$ des tweeenhalftoons. Maer de telder en is gheen 10 000, om die dan daer toe te brenghen ick seg 13 348 gheeft 10 000 wat 10 000? comt 7 491. Inder voughen dat de tweeenhalftoon is van Reden ${}^{10\,000}/_{7\,491}$, de selve ghetrocken van Reden ${}^{10\,000}/_{6\,674}$ des drieenhalftoons blyft (na verandereing in ghemeen telder 10 000) Reden ${}^{10\,000}/_{8\,909}$ des toons tot de selve ghedaen noch alsulck Reden comt Reden ${}^{10\,000}/_{7\,937}$ des tweetoons, de selve van Reden ${}^{10\,000}/_{7\,491}$ des tweeenhalftoons blyft Reden ${}^{10\,000}/_{9\,438}$ des halftoons. Dese toonen bekent wesende, al dander worden openbaer duer verscheyden manieren van wercking want om te hebben de Reden des onderhalftoons men mach trecken den toon vanden tweeenhalftoon ofte den tweetoon vanden 3 ${}^{1}/_{2}$ toon ofte vergaren den toon tot den halftoon, ende alsoo met al d'ander.

*) Dese ghetalen moeten noch eens overzien wesen teghen den oirspronck.

Arithmetical Division of the Monochord

So far we have spoken of the geometrical division; in the following we shall explain the arithmetical division, *i.e.* by simple numbers, which in practice suffice, as follows. I divide the line AB into 10 000 equal parts. Now in order to know how many of these parts pertain to each tone [1]), we begin with the three-tone-and-half, saying: 1 gives $\sqrt{(12)}\ {}^{1}/_{128}$, what does 10 000 give? This makes $\sqrt{(12)}$ 7 812 500 000 000 000 000 000 000 000 000 000 000 000 000 000,
which comes very near to the integer 6 674. For 6 675 is too much, which may appear from the twelfth power of the one and of the other. But to furnish a proper test in each part of the operation of finding these roots, it is to be noted that the square root of the above written fourth power [2]) is nearly $\sqrt{(6)}$ 88 388 347 648 318 440 550 105. Those who wish to check this may multiply this number by itself and add thereto the defect [3]) 96 389 809 968 824 984 488 975. It also appears from the aforesaid defect that the true root exceeds the number by less than one unit, only by about $\frac{96\ 389\ 809\ 968\ 824\ 984\ 488\ 975}{176\ 776\ 695\ 296\ 636\ 881\ 100\ 211}$. When from the aforesaid $\sqrt{(6)}$ 88 388 347 648 318 440 550 105 the square root is again extracted, this makes $\sqrt{(3)}$ 297 301 778 750. The proof is evident by multiplication of this number by itself and addition of the defect 404 488 987 605; it also appears from the aforesaid defect that the true root exceeds the number found by less than one unit, the excess being only about $\frac{404\ 488\ 987\ 605}{594\ 603\ 557\ 501}$ *). From this finally extracting the cube root, we get 6 674, the proof of this being that when 6 674 is raised to the third power and the defect 26 628 726 is added thereto, this makes the first cube number. It also appears from the aforesaid defect that the true cube root exceeds the number found by less than one unit, only by about $\frac{26\ 628\ 726}{133\ 646\ 851}$, so that 6 674 is the nearest integer number to the aforesaid root. The ratio of the three-tone-and-half has to be taken in simple numbers, as from 10 000 to 6 674. Then, this ratio 10 000 : 6 674 of the three-tone-and-half being subtracted from the ratio 2 : 1 of the six-tone, there remains the ratio ${}^{13\,348}/_{10\,000}$ of the two-tone-and-half. But the numerator is not 10 000. In order to bring it to this value, I say: 13 348 gives 10 000, what does 10 000 give? This makes 7 491. Thus the two-tone-and-half has the ratio ${}^{10\,000}/_{7\,491}$. This being subtracted from the ratio ${}^{10\,000}/_{6\,674}$ of the three-tone-and-half, there remains (after conversion to the common numerator 10 000) the ratio ${}^{10\,000}/_{8\,909}$ of the whole tone. Adding to this the same again makes the ratio ${}^{10\,000}/_{7\,937}$ of the ditone. This same subtracted from ratio ${}^{10\,000}/_{7\,491}$ of the two-tone-and-half, there remains the ratio ${}^{10\,000}/_{9\,438}$ of the semitone. These tones being known, all the others are revealed by various methods of operation, for in order to have the ratio of the one-tone-and-half, one may subtract the whole tone from the two-tone-and-half, or the ditone from the three-tone-and-half, or add the whole tone to the semitone, and thus with all others.

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1) Read: interval.

2) The twelfth root of a number is the sixth root of the square root of that number. Stevin is computing the twelfth root, R, of $(10^4)^{12}/2^7 = ((R^3)^2)^2$. First he takes this to be a fourth power ("the above written fourth power"), and he draws a square root, which makes $8.8 \ldots \times 10^{22}$. He again draws a square root, to find $R^3 = 2.97 \ldots \times 10^{11}$. The last step is drawing the third root. Stevin gets 6674.

3) Stevin writes: remainder. In the actual operation of extracting the root this is in fact the remainder after one stops the calculation.

*) These numbers should be compared once more with the original computation.

Men soude de voornoemde deeling oock muegghen aldus doen: Ghevonden hebbende de Reden des drieenhalftoons als boven ick krijgh die des drietoons segghende 1 gheeft $\sqrt{1/2}$ wat 10 000? comt 7 071 inder voughen dat Reden $^{10\,000}/_{7\,071}$ die des drietoons is, de selve ghetrocken van Reden $^{10\,000}/_{6\,674}$ des drieenhalftoons blyft (na verandering in ghemeen telder 10 000) Reden $^{10\,000}/_{9\,438}$ voor den hälftoon, daertoe vergaert noch alsulcken reden comt voor den toon Reden $^{10\,000}/_{8\,908}$ daer wij na deerste manier creghen Reden $^{10\,000}/_{8\,909}$ doirsaeck van welck verschilken openbaer is. Wij sullen ons inde onderschreven tafel om oirdentlicker vervolghs wil an 8 909 houden ende om der ghelycke oirsake inden tweetoon an 7 936. De ghetalen boven den sestoon worden lichtelick ghevonden duer halving der voorgaende, als om te hebben tghetal des sessenhalftoons, ick neem den helft van 9 438 die doet 4 719 ende voor den sevetoon den helft van 8 908 enz. Een sanglijn dan aldus ghedeelt synde, in 10 000 even deelen voor yder toon sullen soo veel deelen commen rekenende van B naer A als de volghende beschryving van dies uytwyst.

10 000.	Selftoon	Eerste
9 438.	Halftoon	Cleen tweede
8 908.	Toon	Groote tweede
8 409.	Onderhalftoon	Cleen derde
7 936.	Tweetoon	Groote derde
7 491.	Tweeenhalftoon	Goe vierde
7 071.	Drietoon	Qua vierde
6 674.	Drieenhalftoon	Vyfde
6 298.	Viertoon	Cleen seste
5 944.	Vierenhalftoon	Groote seste
5 611.	Vyftoon	Cleen sevende
5 296.	Vyfenhalftoon	Groote sevende
5 000.	Sestoon	Dobbeleerste
4 719.	Sessenhalftoon	Dobbelcleen tweede
4 454.	Sevetoon	Dobbelgroot tweede

Soomen nu wilde sien hoe verre de ghedwaelde deelinghen van Pitagoras, Ptolemeus, Bootus ende Zarlinus buyten den wegh waren, men can daer lichtelick toe commen ende haer redens grootste ghetal oock op 10 000 te stellen. Ic neem de Pitagorische diens tafel tot den drieenhalftoon wert beschreven, soodanich is

One might also perform the aforesaid division as follows. Having found the ratio of the three-tone-and-half as above, I get that of the tritone by saying: 1 gives $\sqrt{1/2}$, what does 10 000 give? This makes 7 071, so that the ratio $^{10\,000}/_{7\,071}$ is that of the tritone. This being subtracted from the ratio $^{10\,000}/_{6\,674}$ of the three-tone-and-half, there remains (after conversion to the common numerator 10 000) the ratio $^{10\,000}/_{9\,438}$ for the semitone. Adding the same ratio makes for the whole tone the ratio $^{10\,000}/_{8\,908}$, whereas by the first method we got $^{10\,000}/_{8\,909}$, the cause of which small difference is evident. In the table below, for the sake of a more regular sequence, we shall stick to 8 909, and for the same reason to 7 936 for the ditone. The numbers above the six-tone are easily found by halving the preceding number; thus, to get the number of the six-tone-and-half, I take one half of 9 438, which makes 4 719, and for the seven-tone one half of 8 908 etc. A monochord, therefore, thus being divided into 10 000 equal parts, for every whole tone there will be so many parts, reckoning from B in the direction to A, as the following description shows.

10 000	Selftone	First
9 438	Semitone	Minor second
8 909	Whole tone	Major second
8 409 [1])	One-tone-and-half . . .	Minor third
7 936	Ditone	Major third
7 491	Two-tone-and-half . . .	Good fourth
7 071	Tritone	Bad fourth
6 674	Three-tone-and-half . . .	Fifth
6 298	Four-tone	Minor sixth
5 944	Four-tone-and-half . . .	Major sixth
5 611	Five-tone	Minor seventh
5 296	Five-tone-and-half . . .	Major seventh
5 000	Six-tone	Double-first
4 719	Six-tone-and-half	Double minor second
4 454	Seven-tone	Double major second

If one now wants to see how far amiss were the erroneous divisions of Pythagoras, Boëthius, and Zarlino, this is readily possible by putting the largest number of their ratio also 10 000. I take the Pythagorean division, whose table being described up to the three-tone-and-half, runs as follows:

[1]) In the table a mistake, 8404, has been corrected to 8409. The correct numbers should read:

10 000.0		
9 438.7	7.491.5	5 946.0
8 909.0	7 071.1	5 612.3
8 409.0	6 674.2	5 297.2
7 937.0	6 299.0	5 000.0

10 000	Eerste
9 492	Minste tweede
9 364	Meeste tweede
8 888	Groote tweede
8 437	Cleen derde
7 901	Groote derde
7 500	Goe vierde
7 023	Qua vierde
6 666	Vyfde

Alwaer blyckt dat de cleynste pael des drieenhalftoons van 8 deelen te cort is. Want ghetrocken 6 666 van 6 674 blyft 8 maer den halftoon van 54 deelen te lanck. Ymandt mocht nu dencken waerom dit verschil inden halftoon soo veel grooter is dan inden drieenhalftoon? Daer af seg ick doirsaeck openbaer te wesen int voorbeelt hier boven ghegheven met slechter talen daer wij 110 inde plaets des sestoons stelden ende vyf persoonen A, ... E oirdentlick beteeckenende den drieenhalftoon tweeenhalftoon, toon, tweetoon, ende halftoon, alwaer A inde quad wercking maer een te veel en creegh ende B een te weijnich, C twee te veel D vier te veel maer E vyf te weynich. Inder voughen dat E vyfmael meer te weynich had dan A te veel; Ende even eens uyt de ghelycke oirsaecke cryght hier den halftoon vyfmael meer te weynich (int ansien der Redens vande grofheyt) dan den drieenhalftoon te veel heeft. Uyt desen is oock openbaer dattet verschil des cleen halftoons ende groothalftoons thienmael meerder is dan de Reden des drieenhalftoons te groot ghestelt was, twelck doirsaeck is dat dese dwaling inden halftoon soo veel merckelicker blyckt als in dander toonen.

MERKT

Tis teghedencken dat de namen van de dobbelheyt drievoudicheyt viervoudicheyt der eersten tweeden derden enz. niet en sijn int ansien vande grofheyt der gheluyden maer vande omganghen (nemende acht vervolghende trappen voor een omganck) want ghelyckmen twee drie of vier keeren der slanghens dobbel, drievoudich oft viervoudich mach segghen an een omtrec niet int ansien vande oneven lengden der lynen waer in sulcke Reden niet en bestaet maer opsicht hebbende tot de menichte der keeren: Alsoo heetmen dese eersten tweeden enz. dobbel drievoudich viervoudich int ansien der omganghen sonder te letten opde grofheyt der gheluyden na welck de palen der drievoudicheerste in viervoudighe Reden sijn ende die der viervoudighe eerste in achtvoudighe reden. Inde dobbel eersten overcommet bij ghevalle om wat anders. Want een eersten ofte selftoon te weten Reden $^1/_1$ ghedobbelt dat is daer toe vergaert noch een Reden $^1/_1$ en maeckt al maer Reden $^1/_1$ men heeftse dan alleenlick opsicht tot de omganghen des gheluydts.

10 000	First
9 492	Lesser minor second
9 364	Greater minor second
8 888	Major second
8 437	Minor third
7 901	Major third
7 500	Good fourth
7 023	Bad fourth
6 666	Fifth

From this it appears that the smallest term, of the three-tone-and-half, is 8 parts too short, for when 6 666 is subtracted from 6 674, there remains 8, but the semitone is 54 parts too long. Now one might think: why is this difference so much greater in the semitone than in the three-tone-and-half [1]? I would say that the cause of this is obvious in the example given above in simple language, where we put 110 instead of the six-tone and where five persons, A .... E in due order stood for the three-tone-and-half, the two-tone-and-half, the whole tone, the ditone, and the semitone, where A in the wrong operation received only one too many and B one too few, C two too many, D four too many, but E five too few, so that E had too few five times more than A had too many. And likewise from the same cause the semitone here gets too few, five times more (with respect to the ratio of coarseness) than the three-tone-and-half has too many. From this it is also obvious that the difference between the lesser and the greater semitone is sixteen times [2]) greater than the excess in the ratio of the three-tone-and-half, which is the cause of this error becoming so much more perceptible in the semitone than in the other tones.

NOTE

One must remember that the names of doubleness [3]), triplicity, quadruplicity of firsts, seconds, thirds, etc. do not refer to the coarseness of the sounds, but to the rounds (taking eight [4]) successive steps for a round), for just as one may say that two, three, or four turns of a helix are the double, triple, or quadruple of one circumference, not with respect to the unequal lengths of the lines, in which such a ratio does not consist, but with reference to the number of the turns, so these firsts, seconds, etc. are called double, triple, quadruple with respect to the rounds, without minding the coarseness of the sounds, according to which the constituent notes of the triple first are in the fourfold ratio, and those of the quadruple first in the eightfold ratio. With the double-first the matter happens to be different. For a first or selftone is a ratio 1 : 1. Doubling it means adding another ratio 1 : 1 to it; this makes the ratio 1 : 1 again. Therefore the name double-first refers to the rounds of sound only.

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[1]) Stevin explains why the deviation between his scale and the Pythagorean scale is so much larger for the (minor) semitone (9492 — 9438) than for the fifth (6666 — 6674).

[2]) The Dutch text has: "ten times", obviously by mistake.

[3]) By definition Stevin defined his double-first, our octave. In a double-first (according to the 6th definition) the two notes are in the twofold ratio. This is different from the double of, *i.e.* twice the ratio of the first.

[4]) This should be: "seven". See the 3rd Definition.

ANHANG

Voorreden / Vande Vierde / Van *la si ut* / Vande twelf toonen / De natuer en wort inde compositie niet ghevolcht als in Rhetorica / Der sesten en derden ghelycke daling en climming is wettelick als sij overhandt nu een cleen dan een groote comt / Waerom niet cijferletters inde langhe noten / Bemollaris cantus is onnut onderscheijt. / Tis een ghemeen woordt dat die wel onderscheyt die leert vaec, maer daerbenevens is te weten dat die qualick onderscheyt leert qualick / Species perfecta ende imperfecta sijn al quaet onderscheyt.

Hier vooren beschreven hebbende de spiegheling der Singconst soo heeft mij ghoedt ghedocht daer bij te voughen met corte woorden de verclaring van sommighe duysterheden ende valscheden inde Singdaet deses tydts inghewortelt.

Hooftstick vande vierde

De *) vierde wort vande ghesanckmakers, deses tijts voor quaetluydich ghehouden, alsoo datse in ghesanck met drie of meer stemmen teghen de leeghste niet ghehoort en mach worden ja onder twee stemmen en wilmense gantschelick niet lyden. Maer soomen vraeght waerom? sij antwoorden overmidts datse in ons ghehoir mishaeghlick is: Twelck ick ontken: sal oock de contrari bewijsen eerst met reden daer na, dat meer is, mitterdaet. De reden is dusdanich: Twee gheluyden der dobbeleerste hebben soo grooten ghelijcheyt dat singhende twee persoonen, eenen liedt, ick neem een oudt mensch met een kindt, dese een dobbeleerste hoogher als die doch sonder kennis der dobbeleersten sij en weten ghemeenlick anders niet dan datse beyde in een selfde toon synghen. Ja wij sullen hieroock bewysen dat sulcx dalder ervarendste somtyts ghebeurt. Soo groot dan is dese ghelyckheyt dattet in hen een ghelaet der selfheyt heeft, daerom bij de twee boveschreven stemmen der dobbeleersten ghestelt eenighe derde stem alsulcken aert van soetluidichheyt ofte quaetluijdichheyt als die derde met deene maeckt soodanighe maektse oock met dander. Als neem ick die derde stem een toon boven de leegste wesende, sij maekt daer mee de qualuydighe tweede ende teghen de bovenste de qualuydighe sevende van ghelycke ghedaente: Maer soo de derde stem twee toonen boven de leechste waer, maken de soetluydighe derde ende met de bovenste gheen qualuydighe toon maer de soetluydighe sesten van ghelycke gedaente: Ende vervolghens de derde stem met de bovenste een behaeglick vyfde makende, sij en can met donderste stem niet mishaeghlick wesen, maer maeckt daer teghen een behaeghlicke soetluydighe vierde. Want dit is een ghemeen reghel dat een selfde tot eveneen selfde reden 1 heeft. Hier toe mochtmen noch brenghen de loofweerdiche der Grieken met hun navolghers diet soo mede verstaen hebben, maer die verlatende sullen ant daetlich bewijs commen. Ymandt de vierde quaetluydich achtende segghende die in sijn ghehoir mishaeghlick te wesen, de contrari ende sijn onghelyck wort hun aldus bethoont, men sal nemen eenighe twee verscheyden gheluijden als van een snaer met een menschestem ofte een fluyte met een snaer oft een stem met een fluyte daer mede makende alsnu een vierde alsdan een vyfde ende dat tot verscheydemael ende

*) *De Diatesseron*

APPENDIX [1])

Preface / On the fourth / On *la, si, ut* / On the twelve modes[2]) / In musical composition nature is not followed as it is in Rhetoric / The joint descent and ascent of sixths and thirds is allowed if minor ones alternate with major ones/Why not numerals in the long notes? / *Bemollaris cantus* is a useless distinction. / It is a common saying that whoso distinguishes well teaches well, but besides it must be known that whoso distinguishes improperly, teaches improperly. *Species perfecta* and *imperfecta* is an evil distinction.

Having above described the theory of music, I thought it useful to add, in brief words, an explanation of some obscurities and errors rooted in present-day musical practice.

Chapter on the Fourth)*

The fourth is considered a discord by present-day composers, so that in singing with three or more voices it must not be heard against the lowest part; nay, below two voices it is not suffered at all. But when one asks: why?, they answer: because it displeases our ears. Which I deny, and I shall also prove the contrary, first by argument, next – which is more – in practice.[3])

The argument is as follows. Two sounds making a double-first have so great a similitude that, two persons singing a song – I take an old person and a child, the latter singing a double-first higher than the former, but without knowing about double-firsts – usually for all they know they are singing the same note. Nay, we shall also prove here that this will sometimes happen even to the most experienced people. So great is this similitude, that it has an appearance of identity; therefore, when to the two above-mentioned voices producing the double-first is added a third voice, the same concord or discord that this third voice makes with one of them is also made with the other. If then, for instance, this third voice is a whole tone above the lowest voice, it produces therewith the discordant second and with the higher voice the discordant seventh of a similar character. But if the third voice is two whole tones above the lowest voice, it produces therewith the concord third, and with the highest voice not a discordant tone, but the concordant sixth of a similar character. And consequently, if the third voice produces with the highest voice a pleasant fifth, it cannot be unpleasant with the lowest voice, but produces therewith a pleasant concordant fourth. For this is a common rule that an identical tone is to the same in the ratio 1. To this might also be added the authority of the Greeks and their successors, who also regarded it in this way, but, leaving them alone, we shall come to the empirical proof.

If anyone considers the fourth a discord, saying that it is unpleasant to his hearing, the contrary and his error are proved to him as follows. Take any two different sounds, such as those of a string and a human voice, or a flute and a string, or a voice and a flute, producing therewith now a fourth, now a fifth, and

[1]) Only some of the announced items will be discussed in the following pages.

[2]) A mode consists in the manner in which tones and semitones are distributed within the compass of an octave.

*) The *diatessaron.*

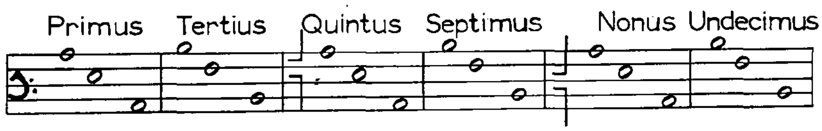
[3]) In a similar argument contained in the other draft Stevin quotes in his favour a treatise of Andreas Papius (1547—1581): ***De consonantiis, seu pro diatesseron libri duo,*** Antverp, 1581, Chrstph. Plantinus.

oock verscheyden vierden en vyfden hoogher en leegher, vraghende telck an syn partie wat het is dat hij sinct ende sullen daetlick bevinden dat hij sonder sekerheyt daer af oirdeelende sijn selven dickwils teghen sal spreken dicmael een vyfde achtende tgene hij te vooren een vierde seyde te wesen ende weder ter contrarie een vierde oirdeelende tgene hij te vooren een vyfde gheseyt had: Twelck soo daetlick blyckende wat behouven wij meer woorden? Wie isser soo onredelick die hem met sijn selfs woorden beschamen sal? segghende de vierde mishaeght mij ende de vyfde bevalt mij seer wel. Maer om deser dynghen oirsake wat breeder te verclaren soo is te weten dat als sulcke twee gheluyden tsamen een eerste ofte dobbel eerste maken dalderscherpste ghehoiren en connen niet sekerlick oirdeelen welck van tween het is. Om hier af by voorbeelt noch opentlicker te spreken ick neem datter twee sijn deen op de fluyte spelende dander synghende, elck sijn partie van eenich liedt ghesonghen ghelyckmen achten soude dattet behoort te wesen. Dit liedt daer naer noch eens overgaen maer alsoo dat den sangher een dobbeleerste hoogher ga dan te vooren, yder (om de reden als vooren te weten datter op een dobbeleerste na gheen sekerheyt en is) hooret voor goedt an, nochtans die toon daer den singher eerst een vyfde onder den fluter was daer sal hij nu nootsaecklick een vierde boven wesen. Inder voughen dat ghenomen het deerste mael een vyfde was soo salmen hier de vierde voor vyfde anhooren. Daerom de ghene die noch segghen dat de vierde in hun ooren mishaeghelick luyt, maer de vyfde seer bevallick, ick en siender niet beter af te besluyten dan dat de ghewoonte uyt de leest eeu ghesproten een weeckheyt in hemlien ghewortelt heeft.

Van de twaelf toonen

Alsoo ick van meijninge was met Meester Davidt te spreken van Sarlijns twaelf thoonen, soo verschreef ick de noten na mijn manier, om hem te bethoonen dattet twaelf waren. Maer alsoo ick vorder meende te bewijsen datter niet meer sijn en conden, bevant ter contrarie datter veertien waren: welck bewijs ick u hier sende. Dus wilt mij uijt den droom helpen, of u selven daer in brengen.

Sarlijns ses thoonen (die met haer contrarie twaelf souden maken) sijn, soo ghijse mij sendt, dusdanich.



this several times and also several fourths and fifths higher and lower, each asking his partner what it is that he is singing or playing, then we shall find in practice that, judging thereof without certainty, he will frequently contradict himself, often regarding as a fifth what he previously stated to be a fourth, and again conversely judging that to be a fourth which he had previously stated to be a fifth. This being found in practice, what more words do we need? Who is so unreasonable as to disgrace himself by his own words, saying: the fourth displeases me and the fifth pleases me very well? But to set forth the cause of these things somewhat more amply, it must be known that when two such sounds produce together a first or a double-first, the very keenest hearing cannot tell for certain which of the two it is. To speak of this even more clearly by way of example, I suppose that there are two persons, one playing the flute and the other singing, each having sung his part of a given song as one would consider it should be. If this song is thereafter repeated again, but in such a way that the singer goes a double-first higher than before, everyone (for the aforesaid reason, to wit that but for a double-first there is no certainty) thinks it is all right. Nevertheless, at the note where the singer first was a fifth below the flutist, he now inevitably will be a fourth above him, in such a manner that, taking that the first time it was a fifth, one now will hear the fourth as a fifth. Therefore, as to those who still say that the fourth sounds unpleasant in their ears, but the fifth very pleasant, I cannot but conclude that the habit grown during the last century has bred a certain effeminacy in them.

On the twelve modes[1]

As I intended to speak with Master David about the twelve modes of Zarlino, I copied the notes in my own way, to show him that there were twelve. But though I further meant to prove that there could not be more, I found on the contrary that there were fourteen, the proof of which I am sending you herewith. Therefore, please undeceive me, or be deceived yourself.

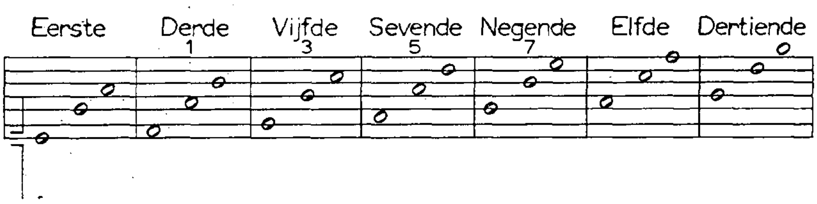
Zarlino's six modes (which would make twelve with their contraries[2]), as you send them to me, are as follows:

Figure 3. The principal notes of the odd numbers of the modes in the Dodekachordon of Glareanus, given to Stevin by a friend who took them from Zarlino.

[1]) In the manuscript a chapter on the twelve modes, as given by Zarlino, is missing. We insert, as a substitute, a copy of a letter from Stevin to an unnamed person, preserved by his son Hendrick. Cf. Note B on p. 461.

[2]) Stevin refers to the *plagal modes* as contrary to the *authentic* modes. By the middle joint note the octave of the mode is divided into a fifth and a fourth. In the authentic modes the fifth is below the fourth, in the plagal modes the fourth is below the fifth. A pair of conjugated authentic and plagal modes, Stevin's *contrary modes*, share their fifths.

Dese stel ick na mijn manier (daer bij voughende de sevende anders de dertiende) aldus



Nu ist kennelick dat uijt de verscheijden plaetsen der halftoonen, onderscheijt wort de verscheijden aert van gesanc diemen verscheijden thoonen noemt. Daerom make ick seven gelijcke fugen in elcken toon een, als hier onder: Mette getippelte trappen beteecken ick tot meerder claerheijt, de plaetsen die op haer voorgaende note halfthoonen maken.

De selve segh ick altemael verscheijden te wezen: twelck ick aldus bewijse:

4^e^.8^e^. Deerste toon haar vierde en achste heeft halfthoonen.

3^e^.7^e^. T'verschil vande derde toon met d'eerste is, dat die haer derde en sevende note halfthoonen heeft dese heeltoonen.

2^e^.6^e^. T'verschil vande vijfde toon met de twee voorgaende is, dat die haer tweede en seste noten halftoonen heeft dese heelthoonen.

5^e^.8^e^. T'verschil vande sevende mette drie voorgaende is, dat die haer vijfde note halfthoon heeft dese heelthoonen; Voorts dat die haer achste note halfthoon heeft, maer de derde en vijfde hebbense heeltoonen.

4^e^.7^e^. T'verschil vande negende toon met de derde vijfde en sevende is, dat die haer vierde note halftoon heeft, maer dese hebbense heeltoonen; Voort dat die haer sevende note halftoon heeft, maer d'eerste vijfde en sevende hebbense heeltoonen.

3^e^.6^e^. T'verschil vande elfde toon met d'eerste, vyfde, sevende en negende is, dat die haer derde note halftoon heeft, dese heeltoonen; Voort dat die haer seste note halftoon heeft, maer de voorgaende eerste, derde, sevende, en negende, hebbense heeltoonen.

2^e^.5^e^. T'verschil vande dertiende toon met d'eerste, derde, sevende, negende en elfde is, dat die haer tweede note halftoon heeft, dese heeltoon: Voort dat die haer vijfde note halftoon heeft, maer d'eerste, derde, vijfde, negende en elfde hebbense heeltoon.

I arrange them in my own way (adding thereto in the seventh place the thirteenth mode), as follows:

Primus	Tertius	Quintus	Septimus	Nonus	Undecimus	
First	Third	Fifth	Seventh	Ninth	Eleventh	Thirteenth

Figure 4. The principal notes of the odd numbers of the modes, supplemented by a thirteenth of Stevin's invention. The small figures indicate the numbers of the ecclesiastical modes.

Now it is evident that it is by the different places of the semitones that the various kinds of music called different modes are distinguished. Therefore I make seven equal scales, one in each mode, as shown below. By the dotted steps I indicate, for greater clarity, the places which make semitones with their preceding notes.

Figure 5. Places of semitones in the various modes, indicated by the dotted lines.

I say that these are all different, which I prove as follows.

4th, 8th. The first mode has its fourth and eighth note making semitones.

3rd, 7th. The difference between the third mode and the first is that the former has its third and seventh notes making semitones, the latter has them making whole tones.

2nd, 6th. The difference between the fifth mode and the two preceding ones is that the former has its second and sixth notes making semitones, in the latter they make whole tones.

5th, 8th. The difference between the seventh mode and the three preceding ones is that the former has its fifth note making a semitone, the latter have it making a whole tone; further that the former has its eighth note making a semitone, but the third and fifth modes have it making a whole tone.

4th, 7th. The difference between the ninth mode and the third, fifth and seventh modes is that the former has its fourth note making a semitone, but the latter have it making a whole tone; further that the former has its seventh note making a semitone, but those of the first, fifth, and seventh modes make whole tones.

3rd, 6th. The difference between the eleventh mode and the first, fifth, seventh, and ninth modes is that the former has its third note making a semitone, the latter have it making a whole tone; further that the former has its sixth note making a semitone, but in the preceding first, third, seventh, and ninth modes it makes a whole tone.

2nd, 5th. The difference between the thirteenth mode and the first, third, seventh, ninth, and eleventh modes is that the former has its second note making a semitone, the other have it making a whole tone; further that the fifth note of the former makes a semitone, but the first, third, fifth, ninth, and eleventh modes have it making a whole tone.

Dese toonen met haer contrarien (welcke contrarien ick om cortheyt achter laet) maecken veerthien toonen, en niet meer en cander wesen, want d'eerstvolgende, twelcke in d'oirden de vijftiende waer, soude sijn als d'eerste, twelck ick bewijsen wilde.

Doirsaeck waerom Zarlin syn eerste toon niet en stelde als dander heeft goede reden overmidts daer duer verdorven soude sijn doirdentlicke voorganck der climming van deen toon tot dander dats van trap tot trap.

Want had hij deerste toon der ouden voor sijn eerste ghenomen soo en soude sijn elfde toon gheen trap hoogher gaen dan sijn voorgaende neghende maer vyf trappen leegher. Ende om de selve reden moet sijn tweede toon oock tot die selve plaets wesen.

Merckt dat ons 1e toon overcomt met haer 10e want sij beyden in *fa* sijn, alleenlick verschillense daer in dat dese hun middelste tsaemval een trap hoogher maeckt dan die. Sgelycx overcomt de 2e met de 11e, de 3e mette 12e, de 5e mette 7e, de 6e mette 8e. Wat de vierde en de 9e belanghen, sij en overcommen noch met malcander noch met eeniche van al dander.

Vant gemeen onderscheijt tusschen de gesanck diemen noemt Bemollaris ende Beduralis

Tgemeen onderscheijt datmen maeckt tusschen de gesanck diemen noemt *Bemollaris* end *Beduralis* is onnut, ende eijgentlick geen onderscheijt, maer al deselfde: Want geeft de C *sol fa ut* sleutel Bemol, den naem van G *sol re ut,* beduyer daer singende, ghij hebt al de selfde gesanck die in Bemol was. Tselve heeft hem alzoo gevende de sleutel van F *fa ut* Bemol, den naem van C *sol fa ut* Bedeur; Sgelijcx de sleutel G *sol re ut* Bemol den naem van D *la sol re* beduijer: Ofte geeft ter contrarie alle dese den naem van die, ende hebt al t'selfde. Ten is dan geen ander aert van gesanck als d'ander en vervolgens soo ist een onut onderscheijt.

Hoofstick waer in doirsaeck verclaert wort vande onvolmaecktheyt dieder int stellen der orghels ende clavesimbels ghebuert

Tgheen wij int voorgaende hooftstick vande vyfde MP gheseyt ende besloten hebben is ghenomen de selfde vyfde MP goet te wesen*) maer tghebuert in

*) Om twelck te bewijsen, soo is te weten dat alsmen syngt (soot derghelicken noemen) *gis* teghen *dis* opwaert als onder anderen Orlando en etc., tzelfde hooren wij een goede vijfde te wesen.

These modes with their contraries (which contraries I leave aside for the sake of brevity) make fourteen modes, and there can be no more, for the next, which would be the fifteenth in the series, would be identical with the first, which I intended to prove.

The motive why Zarlino did not locate his first mode like the others has good reason, since the regular progress of the ascent from one mode to the other, *i.e.* from step to step would have been disturbed by this. For if he had taken the first mode of the Ancients[1]) as his first, his eleventh mode[2]) would not be one step higher than his preceding ninth[3]), but five steps lower. And for the same reason his second tone must also be in the same place.

Note that our first mode corresponds to their tenth, because both are in *fa*.[4]) They only differ in that the latter makes its middle joint note one step lower than the former. Likewise the second corresponds to the eleventh, the third to the twelfth, the fifth to the seventh, the sixth to the eighth. As to the fourth and the ninth they correspond neither to one another nor to any of all the others.

On the Usual Distinction between the Singing that is called Bemollaris and Beduralis.[5])

The usual distinction that is made between the music that is called *Bemollaris* and *Beduralis* is quite useless and is no real distinction, but they are the same thing. For if you give the C *sol fa ut* key, where B is flattened (Bemollaris), the name of G *sol re ut,* singing B natural thereto (Beduralis), you have entirely the same music that was written with B-flat. The same is found when the F *fa ut* key with flattened B is given the name of C *sol fa ut* with B natural, and likewise when the G *sol re ut* key with flattened B is given the name of D *la sol re* with B natural. Or if conversely you give all the latter the name of the former, you will have the same. It is thus no other kind of music than the other, and consequently it is a useless distinction.

Chapter in which is explained the Cause of the Imperfection that arises when Organs and Harpsichords are tuned

What we have said and concluded in a former chapter[6]) about the fifth MP is the finding that this same fifth MP is good,*) but it happens in different in-

1) on *d.*
2) on *c.*
3) on *a.*
4) Stevin here writes *fa* for *c*. This agrees with his nomenclature in figure 1 (p. 436).
5) Cf. Note C on p. 463.
6) See note 1) on p. 437. - Cf. Note D on p. 464.
*) To prove this, it is to be noted that if one sings (as people sometimes call it) *g*-sharp against *d*-sharp upwards, as Orlando, among others, etc., we hear this to be a good fifth.

verscheyden reetschappen soo dervaringh betuycht datse deenmael een weynich te groot valt dandermael een weynich te cleen, somtijts oock goet, maer wantse int spelen niet, oft maer seer weynich ghebruyckt wort, soo latent veel meesters diese stellen, blyven by tgheen tgheval uytbrenght overmidts al de rest diemen besicht sooveel tghehoir belanght, goet ghenouch is. Maer om te verclaren donverclaerden oirsaeck waerom dese vijfde . . . inde voorsz. reetschappen niet soo recht te treffen en is als de natuerlicke ghesanck der menschelicke stemmen die betuijcht te moeten wesen, soo dient verstaen te worden de ghemeene onvolmaecktheijt des werckelicken handels in alle stoffen, welcke niet soo volcommentlick ghelyck de wisconstighe te treffen en is. Als by voorbeelt een stuck lywaet van 50 ellen duer verscheyden persoonen voorsichtelick ghemeten d'een sal een stroobreet ofte duijm meer vinden als dander, Doch soose teenemael ende gants effen uyt commen sonder een haer te verschillen, sulcx ghebuert selden ende by ghevalle, sgelycx heeft hem alsoo int meten der vlacken, lichaemen ende ander stoffen als tijt, roersel, swaerheijten; Oock mede inde stof des gheluyts daer ons verschil af is. Want men can gheen twee gheluyden als der vierde, vyfde of seste etc. alsoo passen dat sijt in duyterste volcommenheyt sijn, ten waer bij ghevalle; daer af oock gheen bewys en can ghedaen worden. Maer om dese werckelicke onvolmaecktheyt duetlick te bethoonen, soo leght den bandt van een luyt ter plaets daer u dunckt haer snaer teghen een ander snaer de volmaeckte vyfde te maken, verschuyf daer naer dien bandt alleenlick soo veel als de dickte van een haer opwaert of neerwaert ende sult bevinden datter gheen merckelicke verandering duer en ghebuert niet teghenstaende datter voor seker eenighe verandering gheschiet. Doch soo ghij vermoedet ende u selven toegaeft die valsheijt der vyfde te bemercken soo laet die verschuijving des bandts duer een ander persoon ghedaen worden, alsoo dat ghij niet en weet of hyse de breede van een haerken opwaert of neerwaer schuyft, ofte op de selve plaets laet; hij u alsoo tot verscheydenmael vraghende na de goetheyt der vyfde sult daetlick u oirdeel onseker bevinden, dicwils de goede quaet segghende ende de quade goet. Tis dan openbaer dattet gheen menschelick ghehoir mueghelick en is hoe scherp het sij twee toonen in haer uyterste volcommenheyt heel seker te passen. Waer uyt volght dat veel sulcke feylen die elck int besonder onbemerckelick sijn, nochtans tsamen een merckelicke dwaling maken; Want ghelyck in tvoorseyde stuck lywaet duer verscheyden persoonen ghemeten veel cleyne verschillekens op yder elle tsamen opt einde eenich merckelick verschil maken, alsoo hier ook inde gheluyden. Want overmidts dese vijfde . . . seer selden ghebruyckt wort soo laetmen die cleyne onbemercklicke feylkens daer op al ancommen welcke ten einde altsamen bemercklick connen sijn, somtijts oock onbemerckelick na tgheval. Daerom en ist gheen wonder dat de bovenste meesters sulcke reetschappen voorsichtelick stellende int laetste nochtans quade toonen ontmoeten die goet behooren te wesen, maer tis natuerlick. Ende diet niet en verstaet hem ghebreeckt de kennis des onderscheyts tusschen werckelicke ende wisconstighen handel, twelck wij bewysen moesten.

struments, as experience shows, that at one time it turns out slightly too large, at another time slightly too small, sometimes also good; but because in playing it is used very little, if at all, many masters who tune the instruments leave it to chance, since all the rest that is used is good enough as regards the ear. But to explain the unexplained cause why this fifth on the aforesaid instruments cannot be hit off as right as the natural singing of human voices testifies it should be, the common imperfection should be understood of practical operation in all matters, which cannot be performed as perfectly as mathematical operations. Thus, for instance, when a piece of linen of 50 yards is carefully measured by different people, one will find a strawbreadth, or an inch more than another. But it seldom happens, and then accidentally, that they all arrive altogether and quite alike at the same result. The same also applies in the measurement of surfaces, solids, and other things, such as time, motion, weight, and again in the matter of sound, which is our point in question. For one cannot match two sounds, such as make a fourth, a fifth or a sixth, etc. in such a way that these intervals are quite perfect, unless by chance; nor can they be proved to be so. But to show this practical imperfection clearly, place the fret of a lute in the place where you think its string will make a perfect fifth against another string. After this, shift this fret over the thickness of a hair only, upwards or downwards, and you will find that no appreciable change takes place, although, to be sure, some change does take place. But if you surmise and concede to yourself that you perceive this falseness of the fifth, let the fret be shifted by someone else, in such a way that you do not know whether he shifts it a hair's breadth upwards or downwards, or leaves it in the same place; when he several times thus inquires of you after the goodness of the fifth, in practice you will find your judgment uncertain, often saying that the good fifth is bad and that the bad fifth is good. It is therefore obvious that no human hearing, however keen it may be, is able to fit two tones quite surely in their perfection. From this it follows that many such mistakes, each of which in itself is inappreciable, yet in combination produce an appreciable error. For as in the aforesaid piece of linen measured by various people many small differences in every single yard, added together, finally make an appreciable difference, so here in the case of sounds too. For since this fifth MP is very rarely used, one let these tiny inappreciable errors drift, which together may finally be appreciable, sometimes also inappreciable, as the case may be. Therefore it is not astonishing that superior masters, tuning these instruments carefully, nevertheless in the end find bad notes which ought to be good; this is but natural. And whoso does not understand this, lacks discrimination between practical and mathematical operations, which we had to prove.

NOTE A, referring to pp. 431/432

In his loose reference to Zarlino, tainted with some derision, Stevin does not do justice to the problem Zarlino was trying to solve. Representing the pitches of the common scale by

$$c \quad d \quad e \quad f \quad g \quad a \quad b \quad c' \quad d' \quad e' \quad \ldots \quad etc.,$$

according to the theory of Ptolemy adopted by Zarlino, in the triad *c, e, g* the note *e* was the harmonic mean between *c* and *g*. The latter making a perfect fifth, *c* and *e* make a perfect major third. The triads *f, a, c'* and *g, b, d'* were transpositions of the same triad. Thus the numbers assigned to these major chords are $24:30:36 = 32:40:48 = 36:45:54$ $(= 4:5:6)$ respectively. This procedure entails a difference of one comma in the whole tones $(d:c)$ and $(e:d)$, with the ratios $^9/_8$ and $^{10}/_9$ respectively; likewise in the whole tones $(g:f)$ and $(a:g)$. It follows that $(a:d)$ is one comma short of a fifth (ratio $^3/_2$). For a transposition of a melody from $c:d:e$ to the initial note *g*, one therefore needs a note α, one comma sharper than *a*. This note can be readily produced in singing, but once an organ pipe or a harpsichord string has been tuned to *a*, it cannot suddenly be brought to α. Zarlino sought for a compromise, by which he might make slight alterations in the pitches that would not disturb the harmonies too much. Suppose we strain the fifth a little bit, by an amount *x*, and the major third by *y*, then the pitches of the common scale will become c, $(d + 2x)$, $(e + y)$, $(f - x)$, $(g + x)$, $(a - x + y)$, $(b + x + y)$, c', $(d' + 2x)$.

We want the new fifths to be equal, therefore

$$(g + x) - c = (a - x + y) - (d + 2x)$$
$$(g - c) - (a - d) = -4x + y = \text{comma}.$$

Likewise there is equality of the new seconds

$$(d + 2x) - c = (e + y) - (d + 2x), \text{ and again}$$
$$(d - c) - (e - d) = y - 4x = \text{comma}.$$

Now, if by a kind of equipartition, the fifth being nearly double the third, one chooses for the strains a relation $x = 2y$, then the solution is $x = -^2/_7$ comma, $y = -^1/_7$ comma. This is the solution which Zarlino offered in his *Institutioni* 1558, Parte II, cap. 43) and to which Stevin refers. Obviously less damage is done to the fundamental intervals if one puts $y = 0$, hence $x = -^1/_4$ comma. This solution was put forward by Zarlino in his *Dimostrationi harmoniche* in 1571 (Ragionamento quinto). Full credit for the first description of this solution is given by Barbour to Pietro Aron (Venice, 1523). Stevin does not mention it. For him the problem does not exist, or rather: he puts $x = -^1/_{12}$ comma, this being the difference between his fifth and the Pythagorean fifth of $^3/_2$.

Therefore the damage done by his equal temperament to the perfect third of $^5/_4$ amounts to $y = ^2/_3$ comma. That solution of the tuning problem was discussed by Zarlino in 1588, in his *Sopplementi musicali* (Libro quarto, cap. 28). Stevin does not mention this at all.

NOTE B, referring to pp. 452/453

In this letter, which is not preserved in his own handwriting, Stevin tentatively enumerates two more, *i.e.* fourteen different modes. In the diagram he indicates by minims (half notes) the principal notes of the odd modes, the so-called authentic modes. These notes are the points of convergence for melodic lines in singing. We believe that Stevin calls *tseamval* (coincidence) the principal note in the middle which joins the fourth and the fifth that constitute the octave. The fact that, against his usual method, he nowhere gives a definition of this term, shows that his work is not complete. The final and initial note of an authentic, odd mode becomes the principle note in the middle of the following even mode, which is called a plagal mode. It turns out that the first and the eighth mode have the same division of the octave, but the difference lies in the positions of the principal notes. In the third diagram[1]), by the added numbers Stevin indicates

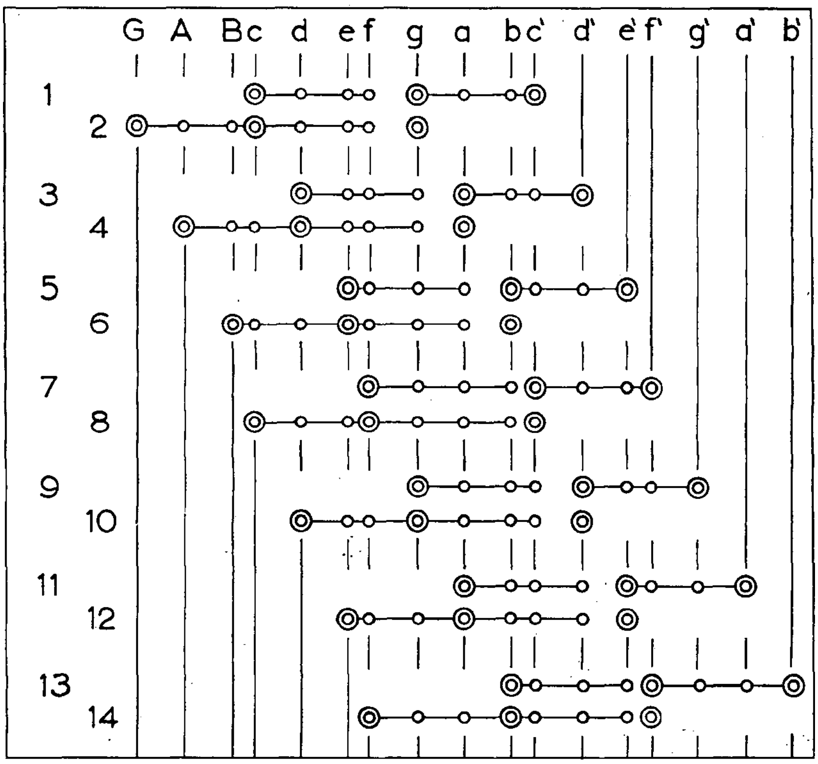


Figure 6. The twelve modes of Zarlino, supplemented with numbers 13 and 14 by Stevin. The latter contain in f : b and in b : f' an augmented fourth and a diminished fifth respectively.

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[1]) Fig. 5 on p. 454.

the similar octaves. For the even mode in question the minim must be put one note lower than for the odd mode. Stevin's thirteenth and fourteenth modes show between the principal notes the discord of a diminished fifth or an augmented fourth. For this reason they had been rejected by Zarlino.
The numbers 1, 3, 5, 7 in Stevin's second diagram refer to the ecclesiastical modes.

In the accompanying diagram the editor gives a full exposition of all these modes (*met haer contrariën*). The editor has also added the title of the chapter and the line beginning with 4e.8e on p. 454.

The last two paragraphs are in Stevin's own handwriting again. He compares his own and Zarlino's numbering of the modes with the numbering of the ecclesiastical modes first. It is not clear what numbering the manuscript refers to in the last paragraph. I keep the numbers as written by Stevin. The correlation between pairs of modes as pointed out by him is found in the pairs (1,8), (2,9), (3,10), (4,11), (5,12), with 6 and 7 solitary.

NOTE C, referring to pp. 456/457

Stevin remarks that one might do without a sign for flat at the beginning of the stave. By a simple transposition, choosing an appropriate clef, a tune can be written without such a flat. He says that if you prescribe a *b*-flat, having *c* on a certain line of the stave, then, by changing the clef you can place *g* on this line. By the same notes on the same lines, without a flat, the same melody comes out as before, written with a *b*-flat.

In a similar way, in figure 1, p. 436, placing *ut* on the key for *g*, *sa* came out on the key for *f*, and there was no need for a black key.

In Stevin's days no importance was attached to absolute pitch. That is the gist of his remark.

The same change of clef transforms the stave line for *f* into a line for *c*, and the line for *g* into a line for *d*.
An example is shown in figure 7.

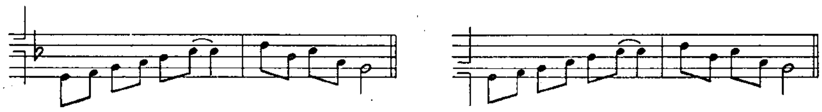

Figure 7. A simple melody: re mi fa sol la sa – ut la sa sol fa –, where sa has been written as b-flat and as f, respectively.

NOTE D, referring to pp. 458/459

This discussion more or less completes the argument in the former chapter on the true ratios of natural tones, pp. 434-435, where the tuning was described of a harpsichord. There the tuning started from F and resulted in a tone H (our A-sharp or B flat) that by ear was to be judged a perfect fifth below F.

The present argument presumes that the tuning has started from K or P (see figure 1, our E-flat), and led to M (our G-sharp). Stevin's contention here is that M and P (G-sharp and *e*-flat) make a perfect fifth. In the note where he refers to other people's meaning, M and P are called *g*-sharp and *d*-sharp. This is the only place where Stevin writes sharps.

It is quite remarkable that Stevin, discussing the possibility of perfect tuning, nowhere mentions that listening to beats of simultaneous sounds offers a powerful means for judging the accuracy of tuning.